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An Analysis of Optimum Pulse Shaping Filter in Time-Discrete Multipath Rayleigh Fading Channels *

マルチパスをともなう通信路における最適なパルス整形フィルタ

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Abstract: This paper concerns with an optimal pulse shaping filter in time-discrete multipath Rayleigh fading environment. The shift orthogonal pulse shaping filter is proposed and proved to be the optimal envelope in the sense of least average bit error probability. It is suited to the Rayleigh fading channel with an arbitrary strength/delay profile of multiple time-discrete paths for BPSK and QPSK signaling etc. Numerical results show that shift orthogonal waveform based pulse shaping filter has lower bit error rate than general shaping waveforms such as square root raised cosine (SRRC) waveform.

Key words: matched filter bound, equally spaced Rayleigh fading channel model, pulse shaping filter, shift orthogonality

1. Introduction

In many data communication systems such as digital mobile radio and terrestrial broadcast systems, data signals are always affected by multipath fading and require some form of channel compensation for the resulting signal distortion. The severe performance degradation effects associated with multipath fading in radio channels are well known. However, the effects of fading can be substantially mitigated through similarly as many well known techniques such as diversity, coded modulation, pilot, and equalization, which have been widely studied and employed. Different methods are adopted to compensate the fading effects in distinct channels, while now, we deal with the issues from another viewpoint, in which the pulse shaping filter design[2)] is concerned once more. Indeed, signal design has an increasing important role to play in wireless communication systems for a host of applications. In this paper, the time shift orthogonal pulse shaping filter is deduced and proved to be an optimal waveform in the sense

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of least average bit error rate in the time-discrete Rayleigh fading channels. The optimal baseband waveform is independent of the path parameters except that the orthogonal shift distance is decided by the path spacing. Therefore, the optimum is a universal principle in time-discrete Rayleigh fading channels.

The organization of the paper is as follows. In section 2, we describe time-discrete multipath Rayleigh fading channels and the lower matched filter bound of the average bit error rate. Section 3 develops the optimal shaping filter and points out that shift orthogonal waveform is an optimal one. In Section 4, the numerical results of the optimum envelope conducted over Pan-European Digital Cellular (GSM) channels such as the typical urban (TU) and hilly terrain (HT) models are presented, in comparison with general square root raised cosine (SRRC) waveforms which is employed in the North America Digital Cellular standard IS-54. Finally, we present the concluding remarks in section 5.

2. Channel model and the lower matched filter bound

We begin with a time-discrete Rayleigh fading channel at baseband which consists of p equally spaced paths. With a baseband model, it can be expressed by an impulse response as 1),2)

$$h(t)=\sum_{i=1}^{p}\alpha_i(t)z_i(t)\delta(t-\tau_i) \qquad (1)$$

where $z_i(t)$ is a slow time-varying, zero mean, unit variance, complex Gaussian random process, τ_i and $\alpha_i(t)$ are the delay and the root mean square (r.m.s.) value of i-th path attenuation coefficient at time t. The delays τ_i are assumed to be constants and equally spaced by $\Delta\tau$ for a particular channel, i.e. $\tau_{i+1}-\tau_i=\Delta\tau$, i=1,…,p-1. Supposing that BPSK is preferred and the transmitted information is denoted as $s_k=\pm1$, the baseband signaling waveform is $s_k g(t)$, $g(t)$ is a shaping pulse. In view of the above description, if the transmitted pulse passed through the channel (1) corrupted by additive white Gaussian noise, the received signal at baseband is

$$r(t)=\sum_{i=1}^{p}\alpha_i z_i g(t-\tau_i)s_k+n(t) \qquad (2)$$

where $n(t)$ is the zero mean, complex white Gaussian random process with power spectral density N_0. We also assume that the slowly varying random process $z_i(t)$ does not change within the duration of the pulse. As we know, the matched filter bound (MFB) is a bound on the optimal performance over a communication channel that assumes that the transmitted pulses are sufficiently separated so that no intersymbol interference occurs[1]. Thus, the MFB is a lower bound, which may not be achievable for specific implementations. However, it has theoretical and practical importance for measuring the limiting performance of receivers under considerations. It is shown in 1) that the lower MFB of average bit error rate can be obtained as follows:

$$P_{ave}(\rho)=\frac{1}{2}\sum_{i=1}^{p}k_i\left[1-\sqrt{\overline{\lambda_i}\rho\Big/\left(1+\overline{\lambda_i}\rho\right)}\right] \qquad (3)$$

where, another variable M has been introduced, M is non-negative definite $p*p$ Hermitian matrix with elements

$m_{ij}=\alpha_i\alpha_j^*R(\tau_i-\tau_j)$, $i,j=1,\cdots,p$,

$R(\bullet)=F^{-1}\left[|G(*)|^2\right]$ is the autocorrelation function of $g(t)$, $G(\omega)$ is the Fourier transform of $g(t)$. We implicitly used the fact that the envelope $g(t)$ is real valued , hence $R(-\tau)=R(\tau)$ and naturally M is a symmetric Hermitian matrix with all elements not less than zero, i.e. $m_{ij}\ge0$. λ_i $(\lambda_i\ge0)$ are the eigenvalues of M and they are assumed to contain only singular ones (no multiple). $\rho=\frac{1}{2N_0}\sum_{i=1}^{p}\lambda_i$ can be proved to be just the averaged received signal to noise ratio, it is assumed to be constant for a given model for the convenience of evaluation.

$$k_i=\prod_{j=1,j\ne i}^{p}\frac{\overline{\lambda_i}}{\overline{\lambda_i}-\overline{\lambda_j}},$$

$\overline{\lambda_i}$ are normalized eigenvalues, i.e.

$$\overline{\lambda_i}=\lambda_i\Big/\sum_{j=1}^{p}\lambda_j=\frac{\lambda_i}{\mathrm{Tr}[M]}.$$

In the following, we will minimize the lower MFB (3) versus eigenvalues of matrix M. Since α_i depends on the channel, and we want to derive an optimum waveform for the Rayleigh fading channel which consists of multiple time-discrete paths with an arbitrary strength/delay profile for BPSK signaling, we have to optimize the objective function with respect to nothing but $g(t)$. By some manipulations, we know that $\sum_{i=1}^{p} k_i = 1$(by integrating equ (11) in 1)) and $\sum_{i=1}^{p} \overline{\lambda_i} = 1$. Hence P_{ave} in (3) can be simplified as

$$P_{ave}(\rho) = \frac{1}{2} - \frac{1}{2}\sum_{i=1}^{p} k_i \left[\sqrt{\overline{\lambda_i}\rho / \left(1+\overline{\lambda_i}\rho\right)}\right] \quad (4)$$

3. Optimum signaling envelope in multipath Rayleigh fading channels

Minimizing $P_{ave}(\rho)$ in (4) is equivalent to maximizing the second item

$$\sum_{i=1}^{p} k_i \left[\sqrt{\overline{\lambda_i}\rho / \left(1+\overline{\lambda_i}\rho\right)}\right].$$

Since it can be deduced that

$$\sum_{i=1}^{p} k_i \left[\sqrt{\overline{\lambda_i}\rho / \left(1+\overline{\lambda_i}\rho\right)}\right] \text{ and } \sum_{i=1}^{p} k_i \left[\overline{\lambda_i}\rho / \left(1+\overline{\lambda_i}\rho\right)\right]$$

(where $k_i = \prod_{j=1, j\neq i}^{p} \frac{\overline{\lambda_i}}{\overline{\lambda_i} - \overline{\lambda_j}}$) are of the same gradient variation trend to individual $\overline{\lambda_i}$ in our situations (i.e., they increase or decrease at the same time to each point $\overline{\lambda_i}$). The goal now can be changed into maximizing the function $P'_{ave}(M) = \sum_{i=1}^{p} k_i \left[\overline{\lambda_i}\rho / \left(1+\overline{\lambda_i}\rho\right)\right]$,

while

$$\begin{aligned} P'_{ave}(M) &= \sum_{i=1}^{p} k_i \left[\overline{\lambda_i}\rho / \left(1+\overline{\lambda_i}\rho\right)\right] \\ &= \sum_{i=1}^{p} \prod_{j=1, j\neq i}^{p} \frac{\overline{\lambda_i}}{\overline{\lambda_i} - \overline{\lambda_j}} \left[\overline{\lambda_i}\rho / \left(1+\overline{\lambda_i}\rho\right)\right] \\ &= \Lambda \qquad (5) \\ &= \frac{\prod_{i=1}^{p}\left(\overline{\lambda_i} + \rho^{-1}\right) - \rho^{-p}}{\prod_{i=1}^{p}\left(\overline{\lambda_i} + \rho^{-1}\right)} = \left(1 + \frac{\rho^{-p}}{Q(M)}\right)^{-1} \end{aligned}$$

where

$$\begin{aligned} &Q(M) \\ &= \prod_{i=1}^{p}\left(\overline{\lambda_i} + \rho^{-1}\right) - \rho^{-p} \\ &= \overline{\lambda_1}\overline{\lambda_2}\Lambda\, \overline{\lambda_p} \\ &+ \rho^{-1}\left(\overline{\lambda_1}\overline{\lambda_2}\Lambda\, \overline{\lambda_{p-1}} + \overline{\lambda_1}\overline{\lambda_3}\overline{\lambda_4}\Lambda\, \overline{\lambda_p} + \Lambda + \overline{\lambda_2}\overline{\lambda_3}\Lambda\, \overline{\lambda_p}\right) \\ &+ \rho^{-2}\left(\overline{\lambda_1}\overline{\lambda_2}\Lambda\, \overline{\lambda_{p-2}} + \overline{\lambda_1}\overline{\lambda_4}\overline{\lambda_5}\Lambda\, \overline{\lambda_p} + \Lambda + \overline{\lambda_3}\overline{\lambda_4}\Lambda\, \overline{\lambda_p}\right) \\ &+ \Lambda + \rho^{-(p-1)}\left(\overline{\lambda_1} + \overline{\lambda_2} + \Lambda + \overline{\lambda_p}\right) \end{aligned}$$

Note that $0 \le \overline{\lambda_i} \le 1, \ \forall i$, which implies that

$$\overline{\lambda_i \lambda_j} \ge \overline{\lambda_i \lambda_j \lambda_k} \ge \Lambda \ge \prod_{l=1}^{p} \overline{\lambda_l},$$

we can obtain that

$$\begin{aligned} &Q(M) \\ &\ge \left(1 + \rho^{-1}C_p^1 + \rho^{-2}C_p^2 + \Lambda + \rho^{-(p-2)}C_p^{p-2}\right) \quad (6) \\ &\prod_{i=1}^{p} \overline{\lambda_i} + \rho^{-(p-1)} \end{aligned}$$

So (6) can be expressed in term of matrix determinant. That is

$$Q(M) \ge g\frac{|M|}{\left[\mathrm{Tr}(M)\right]^p} + \rho^{-(p-1)} \quad (7)$$

where,

$g = 1 + \rho^{-1}C_p^1 + \rho^{-2}C_p^2 + \Lambda + \rho^{-(p-2)}C_p^{p-2} > 0$.

As a result we can obtain that $Q(M)$ increases with the increment of $|M|$ [3].

From the matrix theory 3), we know that the determinant of a non-negative definite matrix with order p arrives maximum when the matrix is diagonal ($|M| \le \prod_{i=1}^{p} m_{ii}$, with equality iff M is diagonal, i.e. $|M| = \prod_{i=1}^{p} m_{ii} \Leftrightarrow m_{ij} = 0, \ \forall$ $i\neq j$), in other word, $R\left(\tau_i - \tau_j\right) = 0$ whenever $i\neq j$. It means that signals from different paths are uncorrelative, and the epoch difference to distinct path equals integer multiple of the path spacing $\Delta\tau$. This is why we call them shift orthogonality.

It is obvious that, without loss of generality, the diagonal matrix

$$\left(|M| = \prod_{i=1}^{p} m_{ii} = \prod_{i=1}^{p} \lambda_i \right)$$

satisfies inequality (7) as well. Furthermore, the diagonal matrix corresponds to the maximized lower bound. Thus, the maximization of lower bound of (6) and (5) has been obtained. Under these conditions, the maximizing bound maximizes the objective itself. The following is to prove that the maximized value of $Q(M)$ in (7) is be of uniqueness [3].

[Gershgorin Theorem] Let $r_i = \sum_{j,j\neq i} m_{ij}$, then its eigenvalue lies within a circle of radius r_i centered at m_{ii}. If some circles overlap, then the overlapping circles contain just as many eigenvalues, counted according to their multiplicity.

The circles condense to points if M is diagonal. So the maximum lower bound corresponds only one set of eigenvalues (There does not exit any other eigenvalues which can make (7) maximum). The solution brings about maximized $P'_{ave}(M)$, and minimization of $P_{ave}(\rho)$ as well. It is evident that the evaluation of optimal baseband waveform corresponding to least BER does not depend on the channel coefficients $\alpha_i(t)$ but determined only by shaping pulse $g(t)$. It is suitable for any equally spaced time-discrete Rayleigh fading channel with arbitrary strength /delay profile. As a result, the THEOREM on shift orthogonality can be stated as,

[**THEOREM**] In the sense of least average bit error probability, the shift orthogonal shaping filter is an optimal baseband waveform over any time-discrete equally spaced multipath Rayleigh fading channel with arbitrary strength/delay profile. Viz, the baseband waveform is required to be orthogonal to its own shifts by integer multiple of the path spacing $\Delta\tau$. ■

4. Numerical results over GSM channels

We consider the GSM TU and HT channel models for both IS-54 signaling and GSM signaling. Although TU and HT models have several variations in the practice, we choose at will a simpler 6-path form model as our examples [1),6)]. The amplitude and delay profiles of the channel models are shown in table 1. We assume that the transmitter shaping filter has a square root raised cosine (SRRC) frequency response with excess bandwidth $\alpha = 0.35$ in the calculation of the average bit error probability in GSM signaling and IS-54 signaling. This kind of SRRC waveform is specified by IS-54. The performance of the shift orthogonal envelope transmitted at the same rate as IS-54 and GSM are also computed over the two GSM channels, in comparison with the aforementioned SRRC shaped IS-54 and GSM signaling.

For IS-54 signaling, although $\pi/4$ QPSK is adopted, we assume that the symbol is transmitted by BPSK for the sake of comparison with other situations. So the bit duration of IS-54 is about 41.152μs. By Jacobi methods, we can get that the normalized eigenvalues for the TU channel and the HT channel shown as Table2.

Table 1. Delay and Amplitude Profile of GSM Channels (profile sampling duration is 0.813us).

	TU model		HT model	
Path #	Delay (us)	Power	Delay (us)	Power
1	0.000	1.000	0.000	1.000
2	0.813	0.669	0.813	0.251
3	1.626	0.448	1.626	0.060
4	2.439	0.300	15.447	0.258
5	3.252	0.200	16.260	0.177
6	4.056	0.134	17.073	0.122

Table 2. The normalized eigenvalues in the TU channel and the HT channel for IS-54 signaling

TU channel for IS-54	HT channel for IS-54
0.99839544	0.96204072
0.00000076	0.00000807
0.00000020	0.00000164
0.00160289	0.03794055
0.00000029	0.00000317
0.00000039	0.00000581

The symbol interval for GSM signaling is 3.692μs. The normalized eigenvalues for GSM TU channel and for the HT shown as Table 3.

We can also attain the results for shift orthogonal waveform. are eigenvalues for TU channel and the HT channel shown as Table 4.

Table 3. The normalized eigenvalues in the TU channel and the HT channel for GSM signaling

TU channel for GSM	HT channel for GSM
0.86080128	0.894089228
0.00094519	0.01018010
0.12472734	0.00010650
0.00028550	0.08765873
0.01306165	0.00019492
0.00017903	0.00777048

Table 4. The shift orthogonal waveform eigenvalues for TU channel and For the HT channel

TU channel for shift orthogonal waveform	HT channel for shift orthogonal waveform
0.55672437	0.84790456
0.24916813	0.05341883
0.11173681	0.00305246
0.05010520	0.02656400
0.02226898	0.02656400
0.02226898	0.01262021

By substituting above eigenvalues into (3) respectively, two curves of average bit error probability versus signal to noise ratio (SNR) over different channels are given in Figure 1 and Figure 2.

Figure 1 shows the performance of waveform shaped by Shift orthogonal and SRRC envelopes over GSM TU channels without any forward correction coding. The comparison of the three curves clearly depicts that the average bit error rate of shift orthogonal waveform is less than the general SRRC ones. The SNR gain is evident especially when SNR is greater than 5dB. The performance comparisons in GSM HT channel also illustrate the advantages of the proposed shift orthogonal waveform, just similar to Figure 1.

Figure 1. Bit error probability curves for shift orthogonal envelope, in comparison with two SRRC waveforms, corresponding to symbol duration of IS-54 and GSM over GSM TU channel.

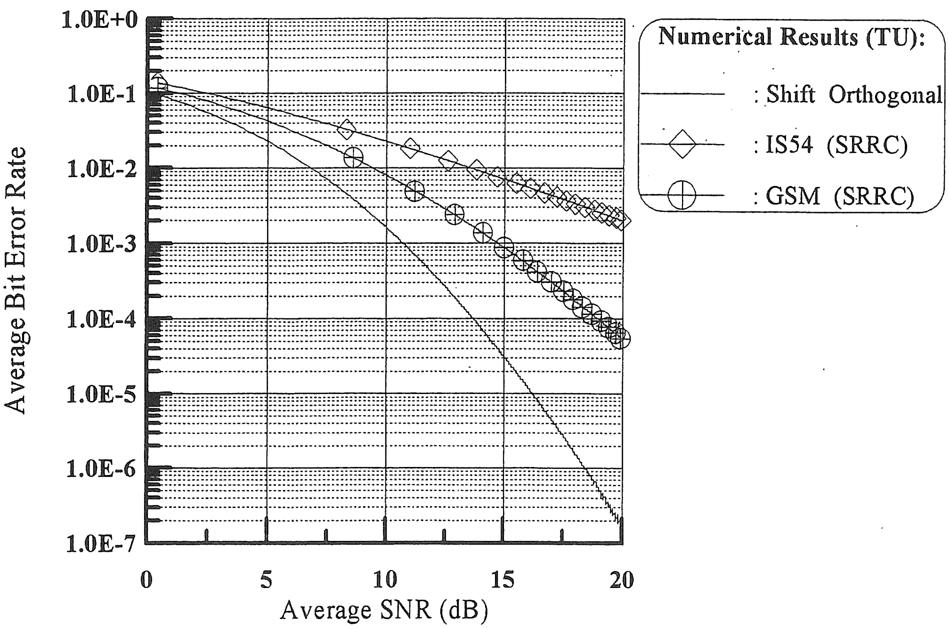

Figure 1. Bit error probability curves for shift orthogonal envelope, in comparison with two SRRC waveforms, corresponding to symbol duration of IS-54 and GSM over GSM TU channel.

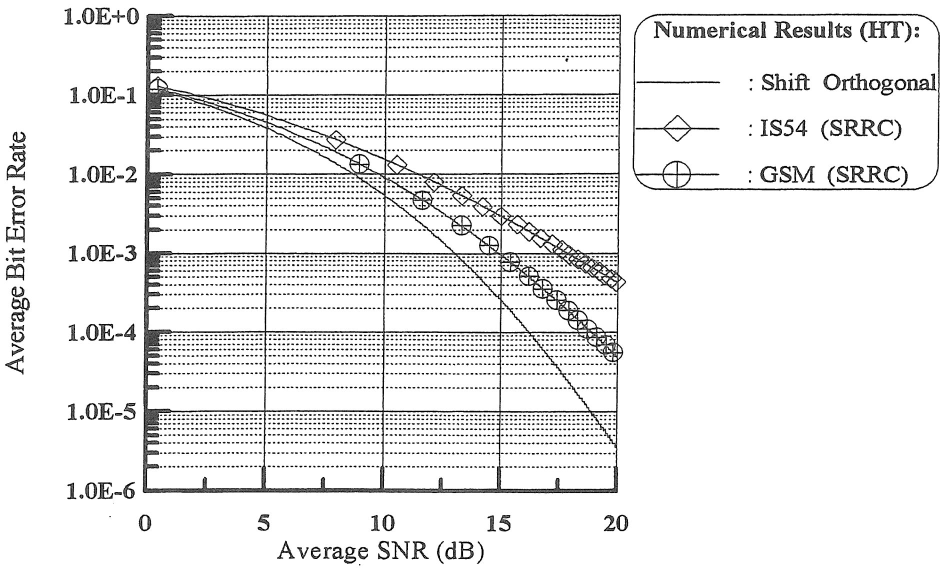

Figure 2. Bit error probability curves for shift orthogonal envelope, in comparison with two SRRC waveforms, corresponding to symbol duration of IS-54 and GSM over GSM HT channel.

5. Concluding Remarks

The optimal baseband signaling envelope based on shift orthogonal envelope for multipath Rayleigh fading channels has been presented in the paper. The numerical results show that the performance is better than general shaping waveforms such as SRRC waveforms, especially in the circumstance when the SNR is larger than 5dB. Since we have the liberty to select a channel, and this avoids the restricting the generality. The results can also be generalized to the cases when the matrix M has multiple roots. Wavelet packet functions [4] are a kind of shift orthogonal functions. It does meet the optimal demands. Furthermore, they are also a special type of Nyquist pulses, so they comply with Nyquist first criterion. However, the major disadvantages of the wavelet functions are their non-constant envelope which would impact the power consumption.

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