

# Application of Specified BCH ECC Decoding Method to diffCDMA with the Continuous Shaping Walsh DS/SS Code

## 連続波形Walsh直接拡散コードを用いた差分CDMAへのBCH誤り訂正符号の特殊符号方式の適用

Kuixi Yin\*, and Masahishi Kishi\*\*

殷奎喜、岸政七

**Abstract** In this paper, the new generator of continuous Walsh code as spread-spectrum code and the new decoding method of BCH code as  $t$ -error correction ( $t \geq 2$ ) code are applied to differential coding CDMA (abbreviated in diffCDMA) system. In this diffCDMA system, the instantaneous phase has already proposed in [1] and [2]. As three important technologies to improve high capacity and conquer the fatal transmission error when communications are carried from high speed running mobiles through such urban environment as rapid multi-ray Rayleigh channel. The error correction technologies are discussed in diffCDMA system with continuous Walsh code from the view points both of mathematical problems for communication theory and electronic circuit implementation

### 1. INTRODUCTION

The typical IMT-2000 is an advance mobile telecommunication system that can provide various high quality services and service terminal at the same time with high intelligence. According to IMT-2000 recommendation, the diffCDMA system has been clearly shown on the developing stages with employing such novel techniques as the continuous chip shaping

secondary modulations, BCH  $t$ -error decoding and correcting, virtual segment interleaving, and etc. [1]. The transmission module of the diffCDMA with both chip continuity and shortened BCH (62, 39) ECC is shown in fig.1. The circuitry skeleton is almost the same to the existing CDMA transmission module only with exception of the two points. The former is employing the shortened BCH ECC circuit pre-fixed to the input terminal. The later is chip continuity circuit CC, which is interpolated between the block of DS/SS and block Wal.

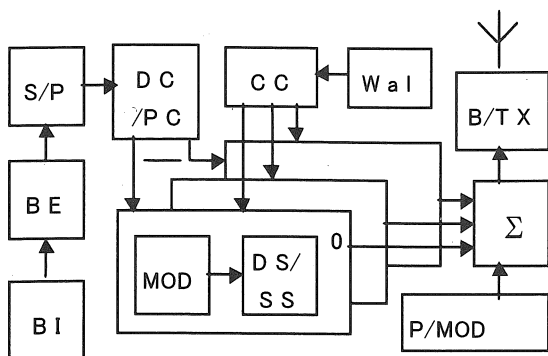


Fig.1 The diffCDMA transmission module with continuity Walsh code and BCH ECC

In fig.1, mark BI, BE, or S/P means the binary information resource, encode of BCH code, or serial-to-parallel converter circuit, respectively. And, mark DC, PC, MOD, or DS/SS means the differential code, phase continuity circuit, the primary modulation, or the direct sequence spread spectrum. Also mark Wal, P/MOD, or B/TX means Walsh spread spectrum code generator, pilot/modulation, or band-pass filter/Transmission unit, respectively.

\*南京師範大学 物理系  
122 Ling Hai Road, Nanjing, 210097 China

\*\*愛知工業大学 総合技術研究所  
Yagusa, Toyota, Aichi, 470-0356 Japan

The total number of diffCDMA

transmission channel is 32 channels, where all of these channels are devoted to carry information with BCH coding. The receiving module scheme of the diffCDMA has shown in reference [2], the total transmission channel number of diffCDMA is 32. In the conventional CDMA, the transmission channel number  $m$  is required to be larger than receiving speech channel  $m'$ , because of the pilot, control, and synchronization are either carried through the redundant  $m-m'$  channels in both cdmaOne and cdma2000.

The computer simulation results are successfully performed to improve two-ray Rayleigh fading of 10dB DUR with 1 micro-second delay spread robustness by vanishing bit errors via employing the enhanced chip shaping, and BCH ( 63, 39 ) at  $E_b/N_0$  of 5db. If communications are carried from 10 km/h walking pedestrians, BER is null at 0 db, if from 300 km / h running vehicles, or vanished at 2.5 db.

## 2. CONCEPT OF ANALYTIC RECEIVING

When system input signal is given by an arbitrary real function  $f(t)$  in the diffCDMA system, the instantaneous phase  $\theta(t)$  and envelope  $A(t)$  are given by eqs.1 and 2 as follows.

$$\theta(t) = \tan^{-1} \frac{\hat{f}(t)}{f(t)} \quad (1)$$

$$A(t) = \sqrt{f^2(t) + \hat{f}^2(t)} \quad (2)$$

The analytic signal  $g(t)$  is, therefore, spanned on the complex plane as shown in eq.3.

$$g(t) = A(t)e^{j\theta(t)} \quad (3)$$

Here,  $\hat{f}(t)$  is Hilbert transform of  $f(t)$ ,  
 $j$  is complex unit,  $j = \sqrt{-1}$ .

The instantaneous phase difference  $\Delta\theta_{ik}(t)$  between two real functions  $f_i(t)$  and  $f_k(t)$  is

strictly derived from a product of corresponding analytic signal  $g_i(t)$  and  $\hat{g}_k(t)$ . Here,  $\hat{g}_k(t)$  is conjugate with the analytic signal  $g_k(t)$  for input  $f_k(t)$ .

$$\begin{aligned} \Delta\theta_{ik}(t) &= \tan^{-1} \{g_i(t)g_k^*(t)\} \\ &= \tan^{-1} [A_i(t)A_k(t) e^{j\{\theta_i(t)-\theta_k(t)\}}] \\ &= \theta_i(t) - \theta_k(t) \end{aligned} \quad (4)$$

The analytic receiving for differential coding PSK has already suggested in equation 4. That is, substituting delayed function  $f_i(t-T)$  into eq.4 instead of  $f_k(t)$ , the equation 4 gives demodulating phase values with most precision time resolution as discussed in equation 5.

$$\Delta\theta(t) = \tan^{-1} \{g(t)g^*(t-T)\} \quad (5)$$

Here,  $T$  is symbol duration.

The equation 5 also shows the other significant results of analytic receiving for detecting in- and quadrature channel signals,  $i(t)$  and  $q(t)$ . That is,

$$\begin{aligned} g(t)g^*(t) &= \{f(t) + j\hat{f}(t)\} \{f(t-T) - j\hat{f}(t-T)\} \\ &= \{f(t)f(t-T) + \hat{f}(t)\hat{f}(t-T)\} \\ &\quad + j\{\hat{f}(t)f(t-T) - f(t)\hat{f}(t-T)\} \end{aligned} \quad (6)$$

$$\begin{cases} i(t) = f(t)f(t-T) + \hat{f}(t)\hat{f}(t-T) \\ q(t) = \hat{f}(t)f(t-T) - f(t)\hat{f}(t-T) \end{cases} \quad (7)$$

According to eqs. from 5 to 7, all functions of  $i(t)$ ,  $q(t)$  and  $\Delta\theta(t)$  are succeeded in excluding carrier frequency, and simultaneously described as full energy form with excluding frequency spectrum elimination. These equations will promise the advantages of tolerance both for frequency shift and multi-ray propagation. The circuitry configuration is directly illustrated for the analytic receiving from eqs.6 and 7 as shown in fig.2, where mark H, D, MUL, or  $\Sigma$  means Hilbert transformer, delay by symbol duration  $T$ , multiplier, or adder, respectively.

The later will yields the most efficient frequency usage compensation function as the absolute solution for differential coding CDMA after de-spreading. The de-spreading in-channel signal  $i_i(t)$  of code- $i$  is given by eq.8 that is is approximately given the averaged within the Walsh de-spreading code duration of  $f_i(t)$ ,  $f_n(t-T)$ ,  $\hat{f}_i(t)$ , or  $\hat{f}_n(t-T)$ , respectively. All of the pilot signals are perfectly excluded in this diffCDMA to yield the most efficient frequency usage.

$$i_i(t) = \left\{ w_i(t) \sum_{l=1}^m w_l(t) f_l(t) \right\} \left\{ w_i(t) \sum_{n=1}^m w_n(t-T) f_n(t-T) \right\} + \left\{ w_i(t) \sum_{l=1}^m w_l(t) \hat{f}_l(t) \right\} \left\{ w_i(t) \sum_{n=1}^m w_n(t-T) \hat{f}_n(t-T) \right\} \quad (8)$$

$$q_i(t) = \left\{ w_i(t) \sum_{l=1}^m w_l(t) \hat{f}_l(t) \right\} \left\{ w_i(t) \sum_{n=1}^m w_n(t-T) f_n(t-T) \right\} - \left\{ w_i(t) \sum_{l=1}^m w_l(t) f_l(t) \right\} \left\{ w_i(t) \sum_{n=1}^m w_n(t-T) \hat{f}_n(t-T) \right\}$$

Here,  $w_l(t)$  is the time  $t$  value of  $l$ -th code.

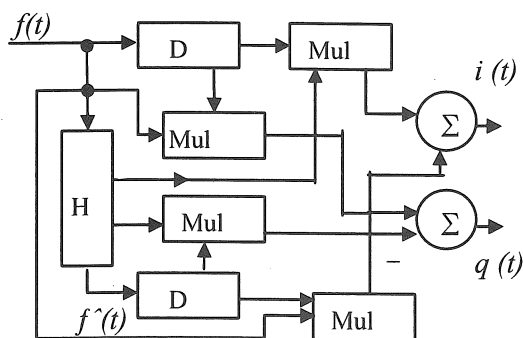


Fig.2 Circuitry configuration of analytic receiving

### 3. NEW generator of continuous Walsh code

The orthogonal Walsh sequence code is also employed in the DS/SS of diffCDMA spread-spectrum communications system. The new generator of continuous Walsh code is divided into generator device and chip continuous device [3]. The new generator of Hardemark sequence matrix is

different from conventional method, for example, Kronecker product, quadratic residue generating method, and etc., which will be facilitated with such significant functional blocks as add 1 counter register, adder| modulus 2, multiplier, and transformer of from 0 to -1. The Hardemark sequence is recognized to be a kind of Walsh sequence matrix.

$$A = [a_{ik}], \quad i = 1, 2, \dots, p, \quad k = 1, 2, \dots, N; N = 2^p \quad (9)$$

$$B = [b_{kj}] = A^T, \quad j = 1, 2, \dots, p \quad k = 1, 2, \dots, N \quad (10)$$

In eqs.9 and 10,  $a_{ik}$  and  $b_{kj}$  is element of +1 counter register made of 0 and 1. The mathematical expression of N by N Hardamark Matrix generator is shown as eq.11 that every sequence code of Hardemark Matrix must be a unique sequence code and must be orthogonal Walsh sequence code, and this allows receivers to distinguish among different user signals.

$$W(p) = A \times B = C = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1N} \\ & & \dots & \\ c_{N1} & c_{N2} & \dots & c_{NN} \end{bmatrix} = C \quad (11)$$

In eq.11,  $c_{ij}$  is a transformed element from Hardemark matrix  $H(p)$ , which represents the digit +1 or -1, and  $C_{ij}$  is transferred to -1 if whose value is  $0 \pmod 2$ , as follows.

$$c_{ij} = \begin{cases} 1, & \text{if } \sum_{k=1}^p a_{ik} b_{kj} \pmod 2 = 1 \\ -1, & \text{if } \sum_{k=1}^p a_{ik} b_{kj} \pmod 2 = 0 \end{cases} \quad (12)$$

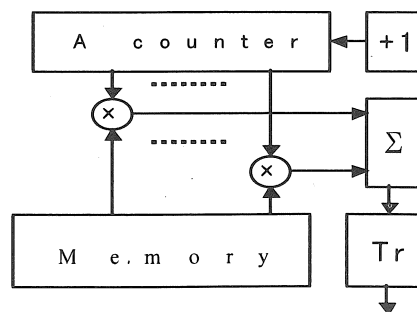


Fig.3 The new generator of Walsh code

Following to from eqs. 9 to 12, the new generator of orthogonal Walsh sequence code is simply implemented as shown in fig.3. In this figure, the mark A counter means a sequential register to get binary number from the 0 to  $N=2^P$  holding eq.9. Memory gives a number defined by the eq.10. Mark +1 mean an adder by 1, and Tr means a transformer to merely change the value from 0 to -1 if  $C_{ij}$  is 0 in modulus 2 as defined by the eq.12. The new generator of orthogonal Walsh sequence code is discussed here to distinguished from existing Walsh sequence generator.

For example, consider about the 4 by 4 Hardemark matrix  $H(2)$ . Values p and N are given by 2 and  $N=2^2=4$ . Matrix A is defined as 4 rows by 2 columns, and matrix B is a transported matrix of A of 2 rows and 4 columns, and matrix C is given as 4 rows and 4 columns, as shown in eqs.13, 14, and 15, respectively.

The orthogonality of Walsh sequence code is shown in fig.4 by its auto-correlation and cross-correlation defined by eq.16. As shown in fig.4,  $Z_k$  show

The chip shaping is effective in diffCDMA as shown in fig.5 for preventing exhaustion from frequency resource in addition to saving transmission power. This chip shaping is also simply facilitated by inserting a

product of 32 by 32 dimensional Walsh matrix, which takes value 32 if  $i$  being equal to  $j$ , and which vanishes if  $i$  being different from  $j$ .

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \quad (13) \quad B = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad (14)$$

$$W(2) = A \times B = \begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 \end{bmatrix} \quad (15)$$

Among corresponding terminals, this orthogonality guarantees that plural pairs of subscribers can span distinguished channels between calling and called parties merely by matching the Walsh sequence code with each other

$$Z_k = \sum_{i=0}^{31} \sum_{j=0}^{31} \overline{H}(5)_{ij} \times \overline{H}(5)_{ij} = \begin{cases} 0, & j \neq i \\ 32, & j = i \end{cases} \quad (16)$$

functional block of chip shaper between DS/SS and Walsh code generator of the diffCDMA system as shown in reference [3].

As shown clearly in fig.5, the spectrum bandwidth of c

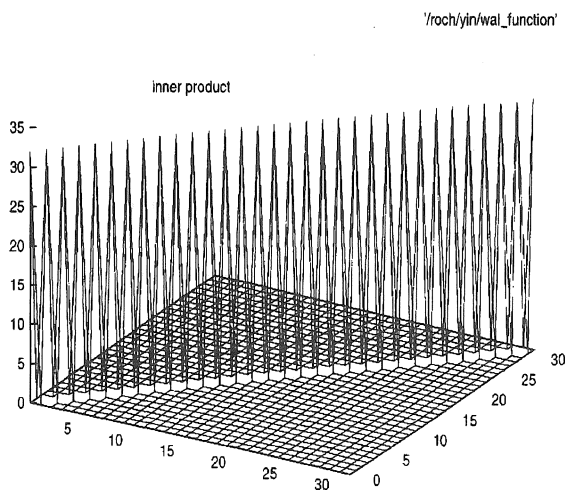


Fig.4. Orthogonality pictorial scheme of Walsh code

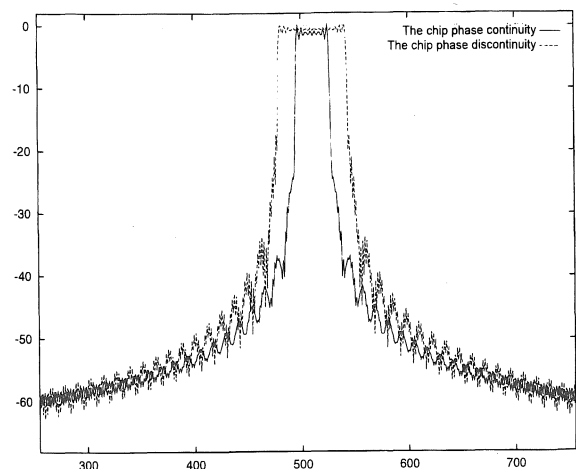


Fig.5 The spectrum occupation comparison between continuous and discontinuous chip shaping of Walsh code

continuous chip shaping is nearly equivalent to a half of spectrum bandwidth of discontinuous chip shape of existing Walsh code.

#### 4. NEW METHOD FOR BCH DECODING

##### 4.1 New Detection Method

Typical IMT-2000 is able to procure the best communication under different transmission conditions. As correcting code of the diffCDMA system, the BCH (63, 39, 4) is employed to correct four errors within 63 bit block along individual bit string of 32k symbol / second [4]. In this paper,  $t$ - errors correction ( $t \geq 2$ ) are also discussed for BCH coding to yield BCH decoding method which is different from both existing Peterson's and other conventional BCH decoding method. This new decoding method does not use the Newton's identities, error location polynomial, nor error evaluation polynomial.

The algebraic structure of BCH code is as well known as defined by the generating polynomial  $G(x)$  as follows. Assuming that  $G(x)$  have eight minimal polynomials,  $M_1(x), M_2(x), M_3(x), \dots, M_8(x)$ , and that these polynomials have the same root of  $GF(2^6)$  in the case of generality, i.e.  $M_1(x)$  and  $M_2(x)$  have the same root. The generating polynomial  $G(x)$  of quadruple error correction BCH code is, therefore, defined as follows.

$$\begin{aligned} G(x) &= LCM\{M_1(x), M_2(x), M_3(x), \dots, M_8(x)\} \\ &= LCM\{M_1(x), M_3(x), M_5(x), M_7(x)\} \\ &= \{(x^6+x+1) \times (x^6+x^4+x^2+x+1) \times \\ &\quad (x^6+x^5+x^2+x+1) \times (x^6+x^3+1)\} \end{aligned} \quad (18)$$

Here,  $LCM$  means the least common multiple  $M_1(x), M_3(x), M_5(x)$  or  $M_7(x)$  is degree 6 minimal polynomial, respectively.

The generating polynomial  $G(x)$  is modified as follows.

$$\begin{aligned} G(x) &= x^{24} + x^{23} + x^{22} + x^{20} + x^{19} + x^{17} + x^{16} + x^{13} \\ &\quad + x^{10} + x^9 + x^8 + x^6 + x^5 + x^4 + x^2 + x + 1 \end{aligned} \quad (19)$$

Generator  $G(x)$  being defined by eq.18 of order 24 polynomial, we can calculate check matrix  $\mathbf{H}$ , which is so called by parity-check matrix, spans by 63 rows and 24 as shown in eq.20.

$$\mathbf{H} = \begin{bmatrix} a^{62} & a^{61} & \dots & a^2 & a^1 & a^0 \\ a^{60} & a^{57} & \dots & a^6 & a^3 & a^0 \\ a^{58} & a^{53} & \dots & a^{10} & a^5 & a^0 \\ a^{56} & a^{49} & \dots & a^{14} & a^7 & a^0 \end{bmatrix} \Big|_{\text{mod}63} \quad (20)$$

Here,  $\alpha$  is a primitive element of  $GF(2^6)$ .

The conventional check matrix  $\mathbf{H}$  is also modified though finite group transform as a new check matrix  $\check{\mathbf{H}}$ , which is a kind of reduced echelon form and expressed by linear combination of the information systematic code form as shown in eq.21.

$$\begin{aligned} \check{\mathbf{H}} &= \begin{bmatrix} \alpha^{11}, \alpha^{17}, \dots, 0, 0, \alpha^5, \alpha^4, \alpha^3, \alpha^2, \alpha^1, \alpha^0 \\ \alpha^{27}, \alpha^{23}, \dots, \alpha^7, \alpha^6, 0, 0, 0, 0, 0, 0 \\ \alpha^{45}, \alpha^{22}, \dots, 0, 0, 0, 0, 0, 0, 0, 0 \\ \alpha^{21}, \alpha^{17}, \dots, 0, 0, 0, 0, 0, 0, 0, 0 \end{bmatrix}^T \\ &= [\mathbf{P}, \mathbf{I}_{24}]^T, \end{aligned} \quad (21)$$

Here,  $\mathbf{P}$  is a parity matrix of 39 rows by 24 columns,  $\mathbf{I}_{24}$  is order 24 identity matrix for tetra-errors correction

This new check matrix  $\check{\mathbf{H}}$  is recognized as a kind of Hamming cod with cyclic shift property. Error will be easily decoded through these cyclic shift and linear combination properties. That is, assuming that the error syndrome being given by a vector,  $\check{\mathbf{H}}$ , if only one digit error exists. If quadruple error exists, the corresponding syndrome  $\check{\mathbf{S}}$  is autonomously given as shown in eq.22 by the summation of the four vectors, which are cyclically shifted  $\check{\mathbf{H}}$  by individual number of error position.

$$\begin{aligned} \check{\mathbf{S}} &= (\mathbf{x} + \mathbf{e}_i + \mathbf{e}_j + \mathbf{e}_k + \mathbf{e}_l) \times \check{\mathbf{H}} \\ &= \mathbf{x} \times \check{\mathbf{H}} + \mathbf{e}_i \times \check{\mathbf{H}} + \mathbf{e}_j \times \check{\mathbf{H}} + \mathbf{e}_k \times \check{\mathbf{H}} + \mathbf{e}_l \times \check{\mathbf{H}} \\ &= 0 + \check{\mathbf{S}}_i + \check{\mathbf{S}}_j + \check{\mathbf{S}}_k + \check{\mathbf{S}}_l \end{aligned}$$

