Studies on Reliability Analysis

for

Microprocessor Systems

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by

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Abstract

This thesis treats several stochastic models of μP systems. Using the theory of Markov renewal processes, the reliability measures such as the mean times to system failure and to completion of the process are obtained. Moreover, the expected costs are derived and optimal policies which minimize them are analytically discussed. Finally, numerical examples of each model are given and some useful discussions are made.

This thesis is divided into 9 chapters. Chapter 1 states fault tolerant techniques and microprocessors (μPs). Chapter 2 considers a μP system with a watchdog timer (WDT) which is preventively maintained at time T and at reset number N. Next, Chapter 3 treats a system where a main processor (MPu) has N watchdog processors (WDPs) with self-checking. To prevent that the MPu becomes faulty, the stochastic model to determine the number of WDPs is formulated. The μP unit which consists of μP and WDP has been recently used. Chapter 4 and Chapter 5 study a system with N μP units. It is assumed in Chapter 4 that a μP is in faulty state if more than K resets have occurred at time T. From the viewpoint of real-time processing of the system, it would be necessary to have the function which completes one processing within a certain limit time. It is assumed in Chapter 5 that a μP is in faulty state if it does not finish one processing until a limit time T. Chapter 6 considers a system with N TMR (Triple Modular Redundancy) units in which each unit consists of μP and WDP. Introducing the concept of complexity, an optimal number of TMR units which minimizes the expected cost is discussed. Chapter 7 deals with the problem for improving the reliability of a μP system with network processing. An optimal policy which minimizes the expected cost until a network processing is successful is discussed. Further, Chapter 8 considers the reliability problem of a μP system whose errors can be detected by using signatures. An optimal division number of a job is discussed. Finally, Chapter 9 summarizes the results derived in this thesis.

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Chapter 1

Introduction

As a computer technology has remarkably developed, microprocessors (μPs) have been used in many practical fields, and the strong demand for improvement of their reliabilities has been increased. However, μPs often fail through some faults due to noise, changes in the environment, hardware errors and programming bugs [Nanya91]. Since μPs have been applied in many systems with high reliability and safety such as automobiles and airplanes, it is imperative that their faults have to be detected and removed rapidly. Recently, several authors have studied and proposed many ideas and devices which watch the behavior of μPs.

Two main approaches to improve the reliability of a system with μPs have been well-known: One is the method of improving the quality of composition elements of the system so that faults, which may cause a system failure, do not occur. This is called *fault avoidance*. The other is the method of weakening the effect of faults by introducing redundant techniques. This is called *fault tolerance*. There are three principal stages in fault tolerant techniques [SS82, Mukaidono88]: Error detection and correction, configuration and recovery, and diagnosis and repair.

In this thesis, we consider the reliability of a μP system from the viewpoint of fault tolerance and concentrates our interest on error detection and correction, and configuration and recovery techniques in fault tolerant systems. System configuration and recovery techniques have a closely mutual relation with the concept of redundant ones in reliability theory. The system is composed of the redundancies of several processors and memories. A high reliability system can be realized by combining these techniques well. Further, to protect faults which may be caused by errors, various kinds of fault tolerant techniques have been used in error detection and correction by a watchdog timer or processor, and the operation of reset.

The theory of Markov renewal processes is used in this thesis to analyze the above stochastic systems: Markov renewal processes were first studied by Lévy (1954) [Lévy54] and Smith (1955) [Smith55]. Pyke (1961a, 1961b) [Pyke61a, Pyke61b] gave a careful definition and discussions in detail. Recently, Çinlar (1975a, 1975b) [Çinlar75a, Çinlar75b] surveyed many results and gave diverse applications in an extensive bibliography. In reliability theory, these processes are one of the most powerful mathematical techniques for analyzing complex systems. Barlow and Proschan (1965) [BP65] gave a table of applicable stochastic processes associated with repairman problems. Further, Nakagawa and Osaki (1979) [NO79] analyzed two-unit systems, using a unique modification of the regeneration point techniques of Markov renewal processes.

This thesis forms several stochastic models of a μP system which reflect actual ones. Using the theory of Markov renewal processes [Osaki92], the reliability measures such as the mean times to system failure and to completion of the process are obtained. Moreover, the expected costs are derived, and optimal policies which minimize them are analytically discussed. Finally, numerical examples of each model are given and some useful discussions are made.

1.1 Fault Tolerant Techniques

Fault tolerant techniques are the method by which a system can realize to tolerate faults, under the condition that they cannot be completely prevented. Actually, as computer systems which have been able to realize the fault tolerance, a multiprocessor system, a dual system and a duplex system have been well-known. In this thesis, some fault tolerant techniques are introduced to improve the reliability of systems.

As a simple method of monitoring the behavior of a μP, a watchdog timer (WDT) has been widely used because it is simple and its cost is low [FN88, NK85]. A WDT can detect some errors of a μP by monitoring periodic signals from a μP. If the signal from a μP does not come within a certain time, a μP is reset by a WDT and the system is recovered. Recently, a WDT with self-checking function has been used in the system because faults of a WDT sometimes occur.

A watchdog processor (WDP) [MM88, Lu82, SM90] is a small and simple coprocessor extending the function of a WDT, and it can detect errors by monitoring the control flow and memory access behavior. For example, an error detection is carried out by memorizing the characteristic information of the monitoring target and by computing the bus information in the operating state, after which results are compared. If its comparison does not agree, a μP is reset by a WDP and the system is recovered.

Generally, when we consider the reliability of a system on an operational stage, we should regard the cause of error occurrences of a μP as faults of software, such as mistakes of operational control and memory access, rather than faults of hardware. That is, when errors of a μP have occurred, it would be effective to recover a system by the operation of reset [Nanya91].

1.2 Microprocessors

The development of fault tolerant techniques relates to the expansion of applicable fields of a μP. In this section, a μP is explained simply.

A CPU (Central Processing Unit) which makes the central part of a computer consists of execution unit and control unit. As IC (Integrated Circuit) technologies have rapidly developed, a CPU has been miniaturized. Such a CPU consists of one chip of LSI (Large Scale Integration) and is called a μP. Moreover, memory systems and input / output devices are connected to a μP [Kaneda91, Ono96, Shima99].

A μP was first produced by Intel Corp. in 1971 and was named Intel4004 [IEICE98]. This μP had 2300 transistors per one chip and had the arithmetic register of 4 bits in length. After that, μPs which have the arithmetic register of 8 bits, 16 bits and 32 bits in length, respectively, were developed. Recently, the μP which has the register of 64 bits in length was produced and its integration level became 15 million transistors per one chip. Thus, μPs become to have advanced functions and high performances. As a result, μPs have been applied in many actual systems such as electrical products, automobiles, communications, and so on.

1.3 Outline of Thesis

This section describes the outline of this thesis. This thesis is divided into Introduction, Chapters 2-8, Conclusions and Bibliography.

Chapter 2 considers a μP system with a WDT which is preventively maintained at time T and at reset number N. The availability of the system is obtained, and an optimal inspection time and reset number which maximize it are analytically discussed. Numerical examples are given when errors of a μP occur according to a Weibull distribution.

Chapter 3 treats a system where a main processor (MPu) has N WDPs with self-checking. If a WDP cannot detect errors of the MPu, the MPu goes to faulty state. To prevent that the MPu becomes faulty, the problem to obtain how many number of standby WDPs is optimal is presented. The reliability function and the expected cost until the main processor becomes faulty are derived, and an optimal number of WDPs which minimizes the expected cost is analytically discussed. Numerical examples are finally given when errors of MPu occur according to an exponential distribution.

The μP unit which consists of μP and WDP has been recently used. Chapter 4 studies a system with N μP units. It is assumed that a μP is in faulty state if more than K resets have occurred at time T. Then, the mean time until system failure is derived. Introducing the cost of a μP, the problem to obtain how many number of μP unit is optimal is analytically discussed. Numerical examples are given when the failure time of a μP is exponential.

From the viewpoint of real-time processing of the system, it would be necessary to have the function which completes one processing within a certain limit time. Chapter 5 discusses the model of a system with N μP units. It is assumed that a μP is in faulty state if it cannot finish one processing until a limit time T. Then, the mean time and the expected processing number until system failure are obtained. Moreover, the cost effectiveness is derived, and an optimal number of μPs which minimizes it is discussed. Numerical examples are finally given for several standard parameters.

Chapter 6 considers a system with N TMR (Triple Modular Redundancy) units in which each unit consists of μP and WDP. Introducing the concept of complexity, the mean time to system failure and the expected cost are derived, and optimal numbers of TMR units which maximize the mean time and minimize the expected cost are analytically discussed. Numerical examples are given when errors of a μP occur according to an exponential distribution.

Chapter 7 deals with the problem for improving the reliability of a μP system with network processing: After the system has made a stand-alone processing, it executes a network communication procedure successively. The mean time and reset number until the success of a network processing are obtained. The expected cost until a network processing is successful is derived, and an optimal reset number which minimizes it is discussed. Numerical examples are given when errors of a μP occur according to an exponential distribution.

Chapter 8 proposes the reliability problem of a μP system whose errors can be detected by using signatures: When a system consists of DMR (Double Modular Redundancy), the same job is executed on two processors and is divided into N tasks with signatures. The mean time and the total processing number of tasks until a job completes successfully are derived. An optimal policy which minimizes the mean time is discussed. Finally, numerical examples are given under suitable conditions.

Finally, Chapter 9 summarizes the results derived in this thesis.

Chapter 2

Optimal Maintenance Policies for a Microprocessor System with Watchdog Timer

This chapter considers a microprocessor (μP) system with a watchdog timer (WDT): When errors of a μP have occurred, a WDT detects them with a certain probability and resets a μP to an initial state. Otherwise, a μP goes to faulty state. To prevent a μP from faults, it is preventively maintained at constant time T or at N-th reset, whichever occurs first. The availability of the system is derived, using the theory of Markov renewal processes. An optimal time T^* and number N^* which maximize the availability are analytically discussed. Finally, numerical examples are given.

2.1 Introduction

As a simple method of monitoring the behavior of a microprocessor (μP), a watchdog timer (WDT) has been widely used because it is simple and its cost is low [FN88, NK85]. A WDT can detect some errors of a μP by monitoring periodic signals from a μP, however, it is impossible to detect any errors. Therefore, it would be necessary to develop a WDT with more advanced capabilities and to improve the reliability of a whole system including a μP.

This chapter considers a μP system with a WDT: When errors of a μP have occurred, a WDT detects them with a certain probability and resets a μP to an initial state. Otherwise, a μP goes to faulty state. To prevent a μP from faults, it is preventively maintained at constant time T or at N-th reset, whichever occurs first. Using the theory of Markov renewal processes [Osaki92], we derive the availability of the system, and discuss analytically an optimal time T^* and number N^* which maximize it. When errors of a μP occur according to a Weibull distribution, numerical examples are given.

2.2 Model and Availability

A WDT monitors the behavior of a μP. When a WDT detects errors of a μP, it resets a μP to an initial state. We assume that:

(1) A WDT works independently of a μP and does not fail.

(2) Errors of a μP occur at a non-homogeneous Poisson process with an intensity function $\lambda(t)$ and a mean-value function $\Lambda(t)$, i.e., $\Lambda(t) \equiv \int_0^t \lambda(u)du$. Hence, the probability that the j-th number of errors have occurred during $(0, t]$ is $H_j(t) \equiv \{[\Lambda(t)]^j/j!\}e^{-\Lambda(t)}(j = 0, 1, 2, \cdots)$.

(3) Errors of a μP can be detected with probability p $(0 < p \leq 1)$ and it is reset to an initial state with probability α $(0 < \alpha \leq 1)$ by a WDT. Otherwise, errors cannot be detected with probability $1-p$ and it is not reset with probability $1-\alpha$. Thus, when errors have occurred, a μP goes to faulty state with probability $1 - p\alpha$.

(4) When a μP has gone to faulty state, it undergoes the corrective maintenance by a user and returns to an initial state according to a general distribution $G_2(t)$ with finite mean $1/\mu_2$.

(5) A μP is inspected and preventively maintained at time T or at N-th reset, whichever occurs first. When time T comes or N-th reset is made before the occurrence of fault, a μP returns to an initial state according to a general distributions $G_3(t)$ with finite mean $1/\mu_3$ or $G_1(t)$ with finite mean $1/\mu_1$, respectively, where $1/\mu_3 < 1/\mu_2$ and $1/\mu_1 < 1/\mu_2$.

We define the following states of the system:

State 0: A μP begins to operate as an initial condition.

State 1: A WDT makes the N-th reset of a μP.

State 2: A μP goes to faulty state.

State 3: The maintenance of a μP begins at time T.

The system states defined above form a Markov renewal process. Transition diagram between system states is shown in Fig.2.1.

Let $Q_{ij}(t)(i,j = 0,1,2,3)$ be one-step transition probabilities of a Markov renewal process. Then, mass functions $Q_{i,j}(t)$ from state i at time 0 to state j at time t are given by

$$
\begin{aligned}
Q_{01}(t) &= (p\alpha)^N \int_0^t \overline{A}(u) H_{N-1}(u) \lambda(u) du, && (2.1)\\
Q_{02}(t) &= \sum_{j=0}^{N-1} (p\alpha)^j (1-p\alpha) \int_0^t \overline{A}(u) H_j(u) \lambda(u) du, && (2.2)\\
Q_{03}(t) &= \sum_{j=0}^{N-1} (p\alpha)^j \int_0^t H_j(u) dA(u), && (2.3)\\
Q_{i0}(t) &= G_i(t) \qquad (i \doteq 1,2,3), && (2.4)
\end{aligned}
$$

where

$$
A(t) \equiv \begin{cases} 1: & t \geq T, \\ 0: & t < T, \end{cases} \tag{2.5}
$$

is the degenerate distribution placing unit mass at T, and $\overline{A}(t) \equiv 1 - A(t)$.

Let $q_{ij}(s)$ and $g_i(s)$ be the Laplace-Stieltjes (LS) transforms of $Q_{ij}(t)$ and $G_i(t)$, respectively. Then, we have

$$
\begin{aligned}
q_{01}(s) &= (p\alpha)^N \int_0^T e^{-st} H_{N-1}(t)\lambda(t)dt, && (2.6)\\
q_{02}(s) &= \sum_{j=0}^{N-1} (p\alpha)^j (1-p\alpha) \int_0^T e^{-st} H_j(t)\lambda(t)dt, && (2.7)\\
q_{03}(s) &= \sum_{j=0}^{N-1} (p\alpha)^j e^{-sT} H_j(T), && (2.8)\\
q_{i0}(s) &= g_i(s) \quad (i = 1, 2, 3). && (2.9)
\end{aligned}
$$

We derive the steady-state availability of the system from (2.6) $\sim$ (2.9). When the system is in state 0 at time 0, the transition probability $P_{00}(t)$ that it is in state 0 at time t is given by

$$
P_{00}(t) = 1 - \sum_{j=1}^{3} Q_{0j}(t) + \sum_{j=1}^{3} Q_{0j}(t) * Q_{j0}(t) * P_{00}(t), \tag{2.10}
$$

where the asterisk mark denotes the Stieltjes convolution. Taking the LS transform of (2.10), we have

$$
p_{00}(s) \equiv \int_0^\infty e^{-st} dP_{00}(t) = \frac{1 - \sum_{j=1}^{3} q_{0j}(s)}{1 - h_{00}(s)}, \tag{2.11}
$$

where

$$
h_{00}(s) \equiv \sum_{j=1}^{3} q_{0j}(s) q_{j0}(s), \tag{2.12}
$$

is the LS transform of the recurrence time distribution to state 0. Thus, the steady-state availability $P(N,T)$ of the system is given by

$$
\begin{aligned}
P(N,T) &\equiv \lim_{t\to\infty} P_{00}(t) = \lim_{s\to 0} p_{00}(s)\\
&= \frac{\sum_{j=0}^{N-1} (p\alpha)^j \int_0^T H_j(t)dt}{\ell_{00}}, && (2.13)
\end{aligned}
$$

where

$$\begin{aligned}\ell_{00} &\equiv \sum_{j=0}^{N-1}(p\alpha)^j\int_0^T H_j(t)dt + \frac{1}{\mu_2} - (\frac{1}{\mu_2} - \frac{1}{\mu_1})(p\alpha)^N \sum_{j=N}^{\infty} H_j(T) \\ &\quad -(\frac{1}{\mu_2} - \frac{1}{\mu_3})\sum_{j=0}^{N-1}(p\alpha)^j H_j(T), \end{aligned} \tag{2.14}$$

is the mean recurrence time to state 0.

In particular, when $T \to \infty$, i.e., a μP is preventively maintained only at N-th reset, the steady-state availability is

$$\begin{aligned}P(N) &\equiv \lim_{T\to\infty} P(N,T) \\ &= \frac{\sum_{j=0}^{N-1}(p\alpha)^j\int_0^\infty H_j(t)dt}{\sum_{j=0}^{N-1}(p\alpha)^j\int_0^\infty H_j(t)dt + \frac{1}{\mu_1}(p\alpha)^N + \frac{1}{\mu_2}[1-(p\alpha)^N]} \end{aligned} \tag{2.15}$$

Similarly, when $N \to \infty$, i.e., a μP is preventively maintained only at time T, the steady-state availability is

$$\begin{aligned}P(T) &\equiv \lim_{N\to\infty} P(N,T) \\ &= \frac{\sum_{j=0}^{\infty}(p\alpha)^j\int_0^T H_j(t)dt}{\sum_{j=0}^{\infty}(p\alpha)^j\int_0^T H_j(t)dt + \frac{1}{\mu_2} - (\frac{1}{\mu_2} - \frac{1}{\mu_3})\sum_{j=0}^{\infty}(p\alpha)^j H_j(T)}. \end{aligned} \tag{2.16}$$

2.3 Optimal Policies

We consider optimal policies which maximize $P(N,T)$ in (2.13) when $\lambda(t)$ is strictly increasing and $\lambda(\infty) \equiv \lim_{t\to\infty}\lambda(t)$.

2.3.1 Optimal reset number

We seek an optimal number N^* which maximizes $P(N,T)$ in (2.13) for a specified T. From the inequality $P(N+1,T) - P(N,T) \leq 0$, we have

$$\sum_{j=0}^{N-1}(p\alpha)^j H_j(T) + (p\alpha)^N \sum_{j=N}^{\infty} H_j(T) + \frac{\sum_{j=N+1}^{\infty} H_j(T)}{\int_0^T H_N(t)dt} \sum_{j=0}^{N-1}(p\alpha)^j(1-p\alpha)\int_0^T H_j(t)dt$$

$$-(1-D)\left[\sum_{j=0}^{N-1}(p\alpha)^j H_j(T) - \frac{H_N(T)}{\int_0^T H_N(t)dt}\sum_{j=0}^{N-1}(p\alpha)^j \int_0^T H_j(t)dt\right] \geq \frac{\frac{1}{\mu_2}}{\frac{1}{\mu_2}-\frac{1}{\mu_1}}, \quad (2.17)$$

where

$$D \equiv \frac{\frac{1}{\mu_2}-\frac{1}{\mu_3}}{\frac{1}{\mu_2}-\frac{1}{\mu_1}}. \quad (2.18)$$

Denote that the left side of (2.17) by $L_T(N)$. Then, when $D \leq 1$, we have $L_T(N) - L_T(N-1) > 0$ from Appendix 2.1, and hence, $L_T(N)$ is strictly increasing in N. It is evident that from the assumption of $1/\mu_1 < 1/\mu_2$,

$$L_T(0) = 1 < \frac{\frac{1}{\mu_2}}{\frac{1}{\mu_2}-\frac{1}{\mu_1}}. \quad (2.19)$$

First, suppose that $D = 1$, i.e., $1/\mu_1 = 1/\mu_3$. Then, since $\lim_{N\to\infty}\sum_{j=N+1}^{\infty} H_j(T)/\int_0^T H_N(t)dt = \lambda(T)$ from the reference [NK83], we have

$$\begin{aligned} L_T(\infty) &\equiv \lim_{N\to\infty} L_T(N) \\ &= e^{-(1-p\alpha)\Lambda(T)} + \lambda(T)(1-p\alpha)\int_0^T e^{-(1-p\alpha)\Lambda(t)}dt. \end{aligned} \quad (2.20)$$

Thus, we have the following optimal policy:

(a) If $L_T(\infty) > (1/\mu_2)/(1/\mu_2 - 1/\mu_1)$, then there exists a finite and unique minimum N^* which satisfies (2.17).

(b) If $L_T(\infty) \le (1/\mu_2)/(1/\mu_2 - 1/\mu_1)$, then $N^* = \infty$, and the steady-state availability is given in (2.16).

Next, suppose that $D < 1$, i.e., $1/\mu_1 < 1/\mu_3$. Then, from Appendix 2.2, we have

$$L_T(\infty) \equiv \lim_{N\to\infty} L_T(N) = \infty. \tag{2.21}$$

Thus, there exists a finite and unique N^* which satisfies (2.17).

When $D > 1$, i.e., $1/\mu_1 > 1/\mu_3$, the mean maintenance time for time T is shorter than that for N-th reset, and hence, we expect that $N^* = \infty$. This obvious fact will be indicated in a numerical example.

2.3.2 Optimal inspection time

We seek an optimal time T^* which maximizes $P(N,T)$ in (2.13) for a specified N. Differentiating equation (2.13) with respect to T and setting it equal to zero, we have

$$\begin{aligned}
&\sum_{j=0}^{N-1}(p\alpha)^j H_j(T) + (p\alpha)^N \sum_{j=N}^{\infty} H_j(T) + \lambda(T)\sum_{j=0}^{N-1}(p\alpha)^j(1-p\alpha)\int_0^T H_j(t)dt \\
&\quad +(D-1)\left[\sum_{j=0}^{N-1}(p\alpha)^j H_j(T) + \lambda(T)\sum_{j=0}^{N-1}(p\alpha)^j(1-p\alpha)\int_0^T H_j(t)dt\right. \\
&\quad \left. +\lambda(T)(p\alpha)^N \sum_{j=0}^{N-1}(p\alpha)^j\int_0^T H_j(t)dt \cdot \frac{H_{N-1}(T)}{\sum_{j=0}^{N-1}(p\alpha)^j H_j(T)}\right] = \frac{\dfrac{1}{\mu_2}}{\dfrac{1}{\mu_2}-\dfrac{1}{\mu_1}}.
\end{aligned} \tag{2.22}$$

Denote that the left side of (2.22) by $L_N(T)$. If $D \ge 1$ and $\lambda(t)$ is strictly increasing, then $L_N(T)$ is also strictly increasing in T from Appendix 2.3 and $L_N(0) = 0$.

First, suppose that $D = 1$. Then, we have,

$$L_N(\infty) \equiv \lim_{T\to\infty} L_N(T)$$

$$= \lambda(\infty)\sum_{j=0}^{N-1}(p\alpha)^j(1-p\alpha)\int_0^\infty H_j(t)dt+(p\alpha)^N. \tag{2.23}$$

Thus, we have the following optimal policy:

(c) If $L_N(\infty) > (1/\mu_2)/(1/\mu_2 - 1/\mu_1)$, i.e.,

$$\lambda(\infty) > \frac{\dfrac{\frac{1}{\mu_2}}{\frac{1}{\mu_2}-\frac{1}{\mu_1}} - (p\alpha)^N}{\sum_{j=0}^{N-1}(p\alpha)^j(1-p\alpha)\int_0^\infty H_j(t)dt},$$

then there exists a finite and unique T^* which satisfies (2.22).

(d) If $L_N(\infty) \leq (1/\mu_2)/(1/\mu_2 - 1/\mu_1)$ then $T^* = \infty$, and the steady-state availability is given in (2.15).

Next, suppose that $D > 1$, i.e., $1/\mu_1 > 1/\mu_3$. Then,

$$L_N(\infty) = D\lambda(\infty)\sum_{j=0}^{N-1}(p\alpha)^j(1-p\alpha)\int_0^\infty H_j(t)dt+(p\alpha)^N. \tag{2.24}$$

Thus, if

$$\lambda(\infty) > \frac{(\frac{1}{\mu_2})[1-(p\alpha)^N]+(\frac{1}{\mu_1})(p\alpha)^N}{(\frac{1}{\mu_2}-\frac{1}{\mu_3})\sum_{j=0}^{N-1}(p\alpha)^j(1-p\alpha)\int_0^\infty H_j(t)dt},$$

then there exists a finite and unique T^* which satisfies (2.22).

If $D < 1$, i.e., $1/\mu_1 < 1/\mu_3$, then it is shown in a numerical example that $T^* = \infty$.

2.4 Numerical Examples

Suppose that $\Lambda(t) = \lambda_0 t^2$, i.e., errors of a μP occur according to a Weibull distribution with shape parameter 2, and its mean time is $\Gamma(1+1/2)/\sqrt{\lambda_0} = 24$ hours. Further, when errors have occurred, the probability that a μP is reset by a WDT is $p\alpha = 0.7 \sim 0.9$.

Table 2.1 gives the numerical example of optimal number N^* when $T = 72, 96, 120, \infty$. When $T = \infty$, this corresponds to the model where a μP is preventively maintained only at N-th reset. When $1/\mu_1 = 1/\mu_3$, it is indicated from Table 2.1 that N^* decreases with T and $(1/\mu_2)/(1/\mu_1)$, however, increases with $p\alpha$. For example, when $p\alpha = 0.8$, $(1/\mu_2)/(1/\mu_1) = 3$ and $T = 120$ hours, the optimal reset number is $N^* = 3$. It is shown that $N^* = \infty$ when $1/\mu_1 > 1/\mu_3$.

Table 2.2 gives the numerical example of optimal time T^* when the upper reset number N is $1, 3, 10, 100, \infty$. When $N = \infty$, this corresponds to the model where a μP is preventively maintained only at time T. When $1/\mu_1 = 1/\mu_3$, it is indicated from Table 2.2 that T^* decreases with N and $(1/\mu_2)/(1/\mu_1)$, however, increases remarkably with $p\alpha$. When $p\alpha$ is small, i.e., the performance facility of a WDT is low, we should maintain a μP at small intervals. It is shown that $T^* = \infty$ when $1/\mu_1 < 1/\mu_3$.

2.5 Conclusions

We have investigated a μP system with WDT which is preventively maintained at time T and at reset number N. We have derived the steady-state availability $P(N, T)$ of the system and have analytically discussed an optimal N^* and T^* which maximize it. From the numerical examples, it has been shown that we have to maintain a μP frequently when the performance facility of a WDT is low. So that, we should make every possible efforts to develop the facilities of a WDT for improving the reliability of the system. When $1/\mu_1 > 1/\mu_3$, i.e., the mean maintenance time for time T is shorter than that for N-th reset, we have to maintain a μP only at time T. Oppositely, when $1/\mu_1 < 1/\mu_3$, we have to maintain a μP only at N-th reset.

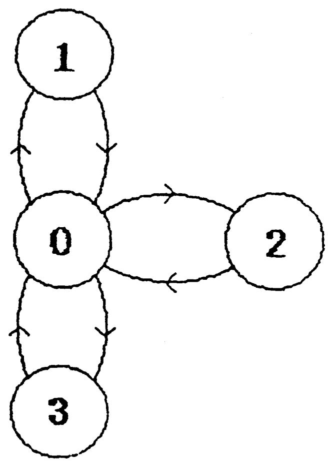


Figure 2.1: Transition diagram between system states.

Table 2.1: Optimal reset number N^*

$p\ \alpha$	T	$(1/\mu_2)/(1/\mu_1) = (1/\mu_2)/(1/\mu_3)$					$(1/\mu_2)/(1/\mu_1) = 1.5$
		2	3	4	5	10	$(1/\mu_2)/(1/\mu_3) = 2$
0.7	72	5	2	2	1	1	∞
	96	4	2	2	1	1	
	120	4	2	2	1	1	
	∞	4	2	2	1	1	
0.8	72	10	3	2	2	1	∞
	96	6	3	2	2	1	
	120	6	3	2	2	1	
	∞	6	3	2	2	1	
0.9	72	37	8	4	3	2	∞
	96	21	6	4	3	2	
	120	13	6	4	3	2	
	∞	12	6	4	3	2	

Table 2.2: Optimal inspection time T^*

p α	N	$(1/\mu_2)/(1/\mu_1) = (1/\mu_2)/(1/\mu_3)$					$(1/\mu_2)/(1/\mu_1) = 2$
		2	3	4	5	10	$(1/\mu_2)/(1/\mu_3) = 1.5$
0.7	1	67	41	32	27	18	∞
	3	55	37	30	26	17	
	10	54	37	30	26	17	
	100	54	37	30	26	17	
	∞	54	37	29	26	17	
0.8	1	92	54	41	34	22	∞
	3	70	46	37	32	21	
	10	67	45	36	31	21	
	100	67	45	36	31	21	
	∞	66	45	36	31	20	
0.9	1	169	92	67	54	32	∞
	3	111	68	53	45	29	
	10	94	64	51	44	29	
	100	94	64	51	44	29	
	∞	93	64	51	44	29	

Appendix

2.1. Proof of $L_T(N) - L_T(N-1) > 0$ when $D \leq 1$

From equation (2.17),

$$L_T(N) - L_T(N-1)$$

$$= \sum_{j=0}^{N-1} (p\alpha)^j (1-p\alpha) \int_0^T H_j(t)dt \left[\frac{\sum_{j=N-1}^{\infty} H_j(T)}{\int_0^T H_N(t)dt} - \frac{\sum_{j=N}^{\infty} H_j(T)}{\int_0^T H_{N-1}(t)dt} \right]$$

$$+(1-D) \sum_{j=0}^{N-1} (p\alpha)^j \int_0^T H_j(t)dt \left[\frac{H_N(T)}{\int_0^T H_N(t)dt} - \frac{H_{N-1}(T)}{\int_0^T H_{N-1}(t)dt} \right] \tag{A2.1}$$

When $\lambda(t)$ is strictly increasing, we have, from the reference [NK83]

$$\frac{\sum_{j=N-1}^{\infty} H_j(T)}{\int_0^T H_N(t)dt} - \frac{\sum_{j=N}^{\infty} H_j(T)}{\int_0^T H_{N-1}(t)dt} > 0. \tag{A2.2}$$

We show only the following inequality:

$$\frac{H_N(T)}{\int_0^T H_N(t)dt} - \frac{H_{N-1}(T)}{\int_0^T H_{N-1}(t)dt} > 0. \tag{A2.3}$$

It is evident that

$$H_N(T) \int_0^T H_{N-1}(t)dt - H_{N-1}(T) \int_0^T H_N(t)dt$$

$$= \frac{H_N(T)}{\Lambda(T)} \left[\Lambda(T) \int_0^T H_{N-1}(t)dt - \int_0^T \Lambda(t) H_{N-1}(t)dt \right]$$

$$= \frac{H_N(T)}{\Lambda(T)} \int_0^T H_{N-1}(t)[\Lambda(T) - \Lambda(t)]dt > 0. \tag{A2.4}$$

Thus, it is proved that $L_T(N) - L_T(N-1) > 0$ when $D \leq 1$ and $\lambda(t)$ is strictly increasing.

2.2. Proof of $L_T(\infty) = \infty$ when $D < 1$

From equation (2.17),

$$L_T(\infty) \equiv \lim_{N\to\infty} L_T(N) = De^{-(1-p\alpha)\Lambda(T)} + \lambda(T)(1-p\alpha)\int_0^T e^{-(1-p\alpha)\Lambda(t)}dt$$

$$+(1-D)\int_0^T e^{-(1-p\alpha)\Lambda(t)}dt\left[\lim_{N\to\infty}\frac{H_N(T)}{\int_0^T H_N(t)dt}\right]. \quad (A2.5)$$

When $\lambda(t)$ is strictly increasing, we easily have

$$\lim_{N\to\infty}\frac{H_N(T)}{\int_0^T H_N(t)dt} = \lim_{N\to\infty}\frac{\frac{[\Lambda(T)]^N}{N!}e^{-\Lambda(T)}}{\int_0^T \frac{[\Lambda(t)]^N}{N!}e^{-\Lambda(t)}dt}$$

$$= \lim_{N\to\infty}\frac{e^{-\Lambda(T)}}{\int_0^T [\frac{\Lambda(t)}{\Lambda(T)}]^N e^{-\Lambda(t)}dt} = \infty. \quad (A2.6)$$

Thus, $L_T(\infty) = \infty$.

2.3. Prove that $L_N(T)$ is increasing in T

When $D \geq 1$ and $\lambda(t)$ is strictly increasing, we show that $L'_N(T) > 0$. From equation (2.22),

$$L'_N(T) = \lambda'(T)\sum_{j=0}^{N-1}(p\alpha)^j(1-p\alpha)\int_0^T H_j(t)dt$$

$$+(D-1)\Bigg\{\lambda'(T)\sum_{j=0}^{N-1}(p\alpha)^j(1-p\alpha)\int_0^T H_j(t)dt$$

$$+\lambda'(T)(p\alpha)^N\sum_{j=0}^{N-1}(p\alpha)^j\int_0^T H_j(t)dt\frac{H_{N-1}(T)}{\sum_{j=0}^{N-1}(p\alpha)^j H_j(T)}$$

$$+\lambda(T)(p\alpha)^N\sum_{j=0}^{N-1}(p\alpha)^j\int_0^T H_j(t)dt$$

$$\times \frac{\lambda(T)}{\sum_{j=0}^{N-1}(p\alpha)^j H_j(T)}\left[H_{N-2}(T)-(p\alpha)H_{N-1}(T)+\frac{(p\alpha)^N[H_{N-1}(T)]^2}{\sum_{j=0}^{N-1}(p\alpha)^j H_j(T)}\right]\Bigg\}. \quad (A2.7)$$

The bracket on the last term in (A2.7) is

$$H_{N-2}(T)-(p\alpha)H_{N-1}(T)+\frac{(p\alpha)^N[H_{N-1}(T)]^2}{\sum_{j=0}^{N-1}(p\alpha)^j H_j(T)}$$

$$= \frac{1}{\sum_{j=0}^{N-1}(p\alpha)^j H_j(T)}\Bigg\{H_{N-2}(T)H_0(T)$$

$$+\sum_{j=1}^{N-1}(p\alpha)^j[H_{N-2}(T)H_j(T)-H_{N-1}(T)H_{j-1}(T)]\Bigg\}. \quad (A2.8)$$

Since

$$H_{N-2}(T)H_j(T)-H_{N-1}(T)H_{j-1}(T)=e^{-2\Lambda(T)}\frac{[\Lambda(T)]^{N+j-2}}{(N-1)!j!}(N-1-j)\geq 0, \quad (A2.9)$$

we have $L'_N(T)>0$. Thus, $L_N(T)$ is strictly increasing when $D\geq 1$ and $\lambda(t)$ is strictly increasing.

Chapter 3

Reliability Evaluations of a Fault Tolerant System with N Watchdog Processors

This chapter considers a fault tolerant system where a main processor (MPu) has N watchdog processors (WDPs) with self-checking: When errors of the MPu have occurred, a WDP detects them with a certain probability and resets the MPu to an initial state. Otherwise, the MPu goes to faulty state. If a WDP fails, it detects the failure with itself and one of other WDPs in standby begins to monitor the MPu again. The above procedures are repeated until all of WDPs have failed. The reliability measures such as the mean time, the reliability and the expected cost until the MPu becomes faulty are derived. An optimal number of WDPs which minimizes the expected cost is analytically discussed. Finally, numerical examples are given.

3.1 Introduction

Generally, microprocessors (μPs) often fail through some faults due to noises and changes in the environment, hardware errors and programming bugs [Nanya91, FN88]. As a simple method of monitoring the behavior of a μP, a watchdog timer (WDT) has been widely used in actual fields [FN88, NK85]. A watchdog processor (WDP)

[MM88, Lu82, SM90] is a small and simple coprocessor, which extends the function of a WDT, and can detect errors by monitoring the control flow and memory access behavior [Nanya91].

This chapter considers a fault tolerant system where a main processor (MPu) has N WDPs with self-checking. The purpose of this model is to improve the reliability of a whole system including the MPu and to derive their reliability measures. If a WDP cannot detect errors of the MPu, the MPu goes to faulty state. Therefore, for prevention that the MPu becomes faulty, we formulate the stochastic model to determine the number of WDPs.

Errors of the MPu occur according to a certain probability distribution and are detected by a WDP. That is, when errors of the MPu have occurred, a WDP detects them with a certain probability, which is called *coverage of a WDP*, and resets the MPu to an initial state. Otherwise, the MPu goes to faulty state. The MPu has N WDPs where one WDP monitors the MPu and the others are in standby. If a WDP fails, it detects the failure with itself and one of other WDPs in standby begins to monitor the MPu again. The above procedures are repeated until all of WDPs have failed. We derive the mean time and the reliability until the MPu becomes faulty. An optimal number of WDPs which minimizes the expected cost is analytically discussed. Finally, numerical examples are given.

3.2 Model and Mean Time

Figure 3.1 draws the outline of the model. We consider the system where a MPu has N standby WDPs and make the following assumptions: A WDP monitors the signature of execution process and judges whether the MPu is normal or abnormal. If a WDP judges that the MPu is abnormal, i.e., a WDP detects errors of the MPu, a WDP resets the MPu to its initial state, although the system cannot determine the cause of error occurrences. That is, the MPu recovers from faulty state by the retrial [Nanya91].

(1) Errors of the MPu due to mistakes of memory access or memory control occur according to a general distribution $F(t)$ with finite mean $1/\lambda$.

(2) A WDP can detect errors of the MPu with probability $p(0 < p \leq 1)$ and resets the MPu to an initial state. This probability p is called *coverage of a WDP*. If a WDP cannot detect errors of the MPu, the MPu goes to faulty state.

(3) Faults of a WDP due to its hardware errors occur according to an exponential distribution $(1 - e^{-\alpha t})$, and a faulty WDP cannot detect any errors of the MPu.

(4) A WDP has self-checking. When faults of a WDP have occurred, it detects them with probability $\theta(0 < \theta \leq 1)$ instantly, and changes to one of standby WDPs. In this case, it resets the MPu to an initial state and begins to monitor the MPu again. On the other hand, if a WDP cannot detect faults of itself with probability $1 - \theta$, a WDP remains in faulty state. In this case, if errors of the MPu occur, it goes to faulty state.

(5) The switch-over from a faulty WDP to a WDP in standby needs a random time according to an exponential distribution $(1 - e^{-\beta t})$ where $\beta > \alpha$. If errors of the MPu occur during the switching, it goes to faulty state.

Under above assumptions, we define the following states of the system:

State i: The i-th WDP begins to monitor the MPu $(i = 1, 2, \cdots, N)$.

State F: The MPu becomes faulty.

The system states defined above form a Markov renewal process [Osaki92] where state F is an absorbing state. Transition diagram between system states is shown in Figure 3.2.

Let $Q_{i,j}(t)(i = 1, 2, \cdots, N; j = 1, 2, \cdots, N, F)$ be one-step transition probabilities of a Markov renewal process and $\phi(s)$ be the Laplace-Stieltjes (LS) transform of any function $\Phi(t)$, i.e., $\phi(s) \equiv \int_0^\infty e^{-st} d\Phi(t)$ for $Re(s) > 0$. Then, from Appendix 3.1, we have

$$q_{i,i}(s) = pf(s+\alpha) \quad (i = 1, 2, \cdots, N), \tag{3.1}$$

$$\begin{aligned} q_{i,F}(s) &= \frac{1}{1-pf(s+\alpha)} \Big\{ (1-p)f(s+\alpha) + (1-\theta)[f(s) - f(s+\alpha)] \\ &\quad + \frac{\alpha\theta}{\alpha-\beta}[f(s+\beta) - f(s+\alpha)] \Big\} \quad (i = 1, 2, \cdots, N), \end{aligned} \tag{3.2}$$

$$\begin{aligned} q_{i,i+1}(s) &= \frac{1}{1-pf(s+\alpha)} \cdot \frac{\alpha\beta\theta}{\alpha-\beta} \Big\{ \frac{1}{s+\beta}[1-f(s+\beta)] \\ &\quad - \frac{1}{s+\alpha}[1-f(s+\alpha)] \Big\} \quad (i = 1, 2, \cdots, N-1), \end{aligned} \tag{3.3}$$

$$q_{N,F}(s) = \frac{1}{1-pf(s+\alpha)}[f(s) - pf(s+\alpha)]. \tag{3.4}$$

We derive the mean time $\ell(N)$ until the MPu becomes faulty. Let $H_N(t)$ be the first-passage time distribution from state 1 to state F. Then, we have

$$H_N(t) = Q_{1,F}(t) + Q_{1,2}(t) * Q_{2,F}(t) + \cdots + Q_{1,2}(t) * \cdots * Q_{N-1,N}(t) * Q_{N,F}(t). \tag{3.5}$$

Taking the LS transforms on both sides of (3.5) and arranging them, we have

$$\begin{aligned} h_N(s) &= \sum_{j=0}^{N-2} \Big\{ \frac{\alpha\beta\theta}{\alpha-\beta} \cdot \frac{1}{1-pf(s+\alpha)} [\frac{1-f(s+\beta)}{s+\beta} - \frac{1-f(s+\alpha)}{s+\alpha}] \Big\}^j \\ &\quad \times \frac{1}{1-pf(s+\alpha)} \Big\{ (1-p)f(s+\alpha) + (1-\theta)[f(s) - f(s+\alpha)] \\ &\quad + \frac{\alpha\theta}{\alpha-\beta}[f(s+\beta) - f(s+\alpha)] \Big\} \\ &\quad + \Big\{ \frac{\alpha\beta\theta}{\alpha-\beta} \cdot \frac{1}{1-pf(s+\alpha)} [\frac{1-f(s+\beta)}{s+\beta} - \frac{1-f(s+\alpha)}{s+\alpha}] \Big\}^{N-1} \\ &\quad \times \frac{1}{1-pf(s+\alpha)}[f(s) - pf(s+\alpha)] \quad (N = 1, 2, \cdots), \end{aligned} \tag{3.6}$$

where $\sum_{j=0}^{-1} \equiv 0$. Hence, the mean time $\ell(N)$ until the MPu becomes faulty is given by

$$\ell(N) \equiv \int_0^\infty t dH_N(t) = \lim_{s \to 0} \frac{d}{ds}[-h_N(s)]$$

$$= \frac{1}{1-pf(\alpha)}\left(\frac{1-A^{N-1}}{1-A}B + \frac{1}{\lambda}A^{N-1}\right) \qquad (N=1,2,\cdots), \tag{3.7}$$

where

$$A \equiv \frac{\theta}{\alpha-\beta}\cdot\frac{\alpha[1-f(\beta)]-\beta[1-f(\alpha)]}{1-pf(\alpha)}, \tag{3.8}$$

$$B \equiv \frac{\theta}{\alpha-\beta}\cdot\left\{\frac{\alpha[1-f(\beta)]}{\beta} - \frac{\beta[1-f(\alpha)]}{\alpha}\right\} + \frac{1}{\lambda}(1-\theta), \tag{3.9}$$

and from $A = q_{i,i+1}(0)$, note that $0 < A < 1$. It is evident that when $N = 0$, i.e., the MPu does not have WDP, we have $h_0(s) = f(s)$ and $\ell(0) = 1/\lambda$.

3.3 Analysis of Reliability

Let $R_N(t)$ be the probability that the MPu does not become faulty until time t and we define $R_N(t) \equiv 1 - H_N(t)$. That is, $R_N(t)$ denotes the reliability function when the MPu has N WDPs.

Especially, suppose that faults occur at random, i.e., $F(t) = 1 - e^{-\lambda t}$. Then, from Appendix 3.2, equation (3.6) is simplified as follows:

$$h_N(s) = \frac{\lambda}{s+\lambda}\left[1 - \frac{sp}{s+\alpha+\lambda(1-p)}\sum_{j=0}^{N-1}\left\{\frac{\alpha\beta\theta}{[s+\alpha+\lambda(1-p)](s+\beta+\lambda)}\right\}^j\right]$$
$$(N=1,2,\cdots). \tag{3.10}$$

Thus, taking the LS inverse transform of $h_N(s)$, from Appendix 3.3, we obtain $R_N(t)$ $(N = 0,1,2,\cdots)$ successively. For example, $R_1(t)$ is given by

$$R_1(t) = e^{-\lambda t} - \frac{p\lambda}{\alpha-\lambda p}\{e^{-[\alpha+\lambda(1-p)]t} - e^{-\lambda t}\}. \tag{3.11}$$

In particular, when $N \to \infty$, from Appendix 3.4, we have

$$\begin{aligned} R_\infty(t) &\equiv \lim_{N\to\infty} R_N(t) \\ &= e^{-\lambda t} + p\lambda\left[\frac{\beta+\lambda-w_1}{(w_1-w_2)(w_1-\lambda)}e^{-w_1 t} - \frac{\beta+\lambda-w_2}{(w_1-w_2)(w_2-\lambda)}e^{-w_2 t}\right. \\ &\quad \left. + \frac{\beta}{(w_1-\lambda)(w_2-\lambda)}e^{-\lambda t}\right], \end{aligned} \tag{3.12}$$

where

$$w_1 \equiv \frac{1}{2}[\alpha+\beta+\lambda(2-p)+\sqrt{(\alpha-\beta-\lambda p)^2+4\alpha\beta\theta}], \tag{3.13}$$

$$w_2 \equiv \frac{1}{2}[\alpha+\beta+\lambda(2-p)-\sqrt{(\alpha-\beta-\lambda p)^2+4\alpha\beta\theta}]. \tag{3.14}$$

3.4 Optimal Policy

Let c_1 be the acquisition cost for a WDP and c_2 be the cost for the fault of the MPu. Then, the expected cost $C(N)$ per unit of time of the system with N WDPs is given by

$$\begin{aligned} C(N) &\equiv \frac{Nc_1+c_2}{\ell(N)} \\ &= \frac{Nc_1+c_2}{\dfrac{1}{1-pf(\alpha)}\left(\dfrac{1-A^{N-1}}{1-A}B+\dfrac{1}{\lambda}A^{N-1}\right)} \quad (N=1,2,\cdots). \end{aligned} \tag{3.15}$$

We seek an optimal number N^* which minimizes $C(N)$ in (3.15). From the inequality $C(N+1)-C(N)\geq 0$, we have

$$\frac{\dfrac{B}{1-A}(1-A^{N-1})+\dfrac{1}{\lambda}A^{N-1}}{A^{N-1}(1-A)}-(N-1)\left(\frac{B}{1-A}-\frac{1}{\lambda}\right)\geq\frac{c_1+c_2}{c_1}\left(\frac{B}{1-A}-\frac{1}{\lambda}\right). \tag{3.16}$$

Hence, if $B/(1-A)-1/\lambda\leq 0$, i.e., $\lambda B\leq 1-A$ then $C(N)$ is strictly increasing in N. In this case, $N^*=0$.

Next, assume that $\lambda B>1-A$. Then, arranging inequality (3.16), we have

$$\frac{1-A^{N-1}+D}{A^{N-1}(1-A)}-(N-1)\geq\frac{c_1+c_2}{c_1}, \tag{3.17}$$

where

$$D\equiv\frac{1-A}{\lambda B-(1-A)}. \tag{3.18}$$

Denoting the left side of (3.17) by $L(N)$, we have

$$L(N)-L(N-1) = \frac{1-A^{N-1}+D}{A^{N-1}}>0, \tag{3.19}$$

$$L(1) = \frac{1}{\lambda B - (1-A)}, \tag{3.20}$$
$$L(\infty) \equiv \lim_{N\to\infty} L(N) = \infty. \tag{3.21}$$

Hence, $L(N)$ is strictly increasing in N from $L(1)$ to ∞. Thus, if $L(1) < (c_1 + c_2)/c_1$, i.e., $\lambda B - (1 - A) > c_1/(c_1 + c_2)$ then there exists $N^*(> 1)$ which satisfies (3.17). Otherwise, if $L(1) \geq (c_1 + c_2)/c_1$, i.e., $\lambda B - (1 - A) \leq c_1/(c_1 + c_2)$ then $N^* = 1$.

Thus, we have the following optimal policy:

(i) If $\lambda B - (1 - A) \leq 0$, then $N^* = 0$ and the expected cost $C(0) = \lambda c_2$. In this case, the MPu should have no WDP.

(ii) If $0 < \lambda B - (1 - A) \leq c_1/(c_1 + c_2)$, then $N^* = 1$.

(iii) If $\lambda B - (1 - A) > c_1/(c_1 + c_2)$, then there exists a finite and unique minimum $N^*(> 1)$ which satisfies (3.17).

3.5 Numerical Examples

We compute numerically the reliability $R_N(t)$ and the optimal number N^* which minimizes $C(N)$. Suppose that errors of the MPu occur according to an exponential distribution $F(t) = 1 - e^{-\lambda t}$. Let the mean hung-up time (1 day $\sim$ 10 days) of a μP correspond to the mean time $1/\lambda$ to error occurrences of the MPu and $1/\lambda = 1$ (day). Let the mean time (1 month $\sim$ 1 year) to error occurrences of a WDT correspond to the mean time $1/\alpha$ to error occurrences of a WDP and $1/\alpha = 30 \sim 365$ (days). Further, for the sake of convenience, suppose that the mean processing time of the switching from a WDP to other WDPs in standby is $1/\beta = 1/(30 \times 10^4)$. Moreover, the probability that a WDP detects the failure with itself is $\theta = 0.8 \sim 0.99$ and the coverage of a WDP is $p = 0.8 \sim 0.99$, the acquisition cost c_1 for a WDP is a unit of cost and the cost rate of the fault of the MPu to a WDP is $c_2/c_1 = 10^2 \sim 10^7$.

Figure 3.3 draws the reliability $R_N(t)$ for $N = 0, 1, 2, 3, 4, \infty$ when $1/\alpha = 30$ (days), $p = 0.99$ and $\theta = 0.8$. This indicates that $R_N(t)$ increases evidently with N. When

$N \geq 1$, $R_N(t)$ increases noticeably compared with the case of $N = 0$, i.e., the MPu does not have a WDP, however, its increase rate decreases gradually with N and nearly converges to the value of $R_\infty(t)$. From this numerical example, it is estimated that the system is enough to have about 3 WDPs.

Next, Table 3.1 gives the optimal number N^* which minimizes $C(N)$ when $1/\lambda = 1$ (day), $p = 0.8$ and $\theta = 0.8$. This indicates that N^* decreases with $1/\alpha$, however, increases with c_2/c_1. For example, when $1/\alpha = 180$ (days) and $c_2/c_1 = 10^3$, the optimal number of WDPs is $N^* = 2$.

Table 3.2 gives the numerical values for the mean time $\ell(N^*)$ until the MPu becomes faulty when $1/\lambda = 1$ day and $1/\alpha = 180$ days. This indicates that $\ell(N^*)$ increases with c_2/c_1, p and θ. It is easily seen that the coverage p gives a greater influence on the mean time than θ. Hence, to develop the reliability of the MPu, we should improve the coverage of a WDP.

3.6 Conclusions

Recently, several authors have studied and proposed many ideas for the improvement of the reliability of the MPu. We have investigated the system where one WDP monitors the behavior of the MPu and the others are in standby. We have derived the mean time until the MPu becomes faulty and the reliability function by considering the mean times to error occurrences of the MPu and WDP, the coverage of a WDP and so on. Further, we have discussed an optimal number of WDPs which minimizes the expected cost.

From the numerical example of the reliability function, it has been shown that it is effective to have at least one WDP when the system requires a high reliability. Further, the optimal number which minimizes the expected cost decreases with $1/\alpha$, however, increases with c_2/c_1. Further, the coverage of a WDP gives a great influence on the improvement of the system.

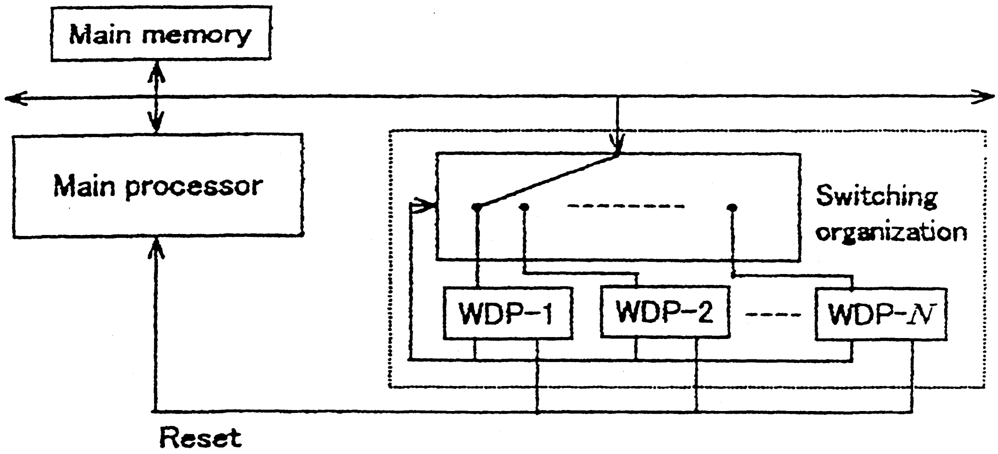


Figure 3.1: Outline of the model with N watchdog processors.

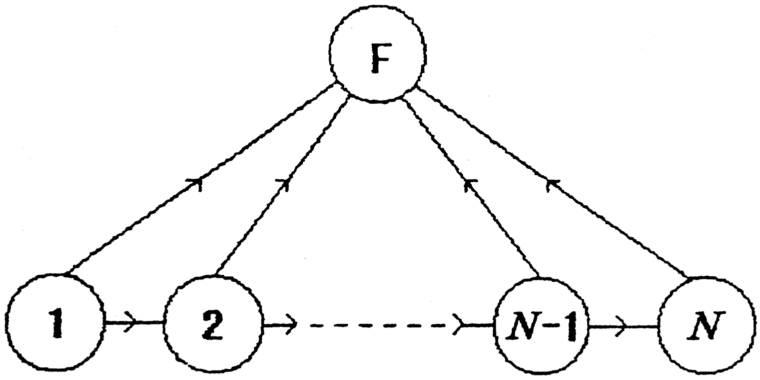


Figure 3.2: Transition diagram between system states.

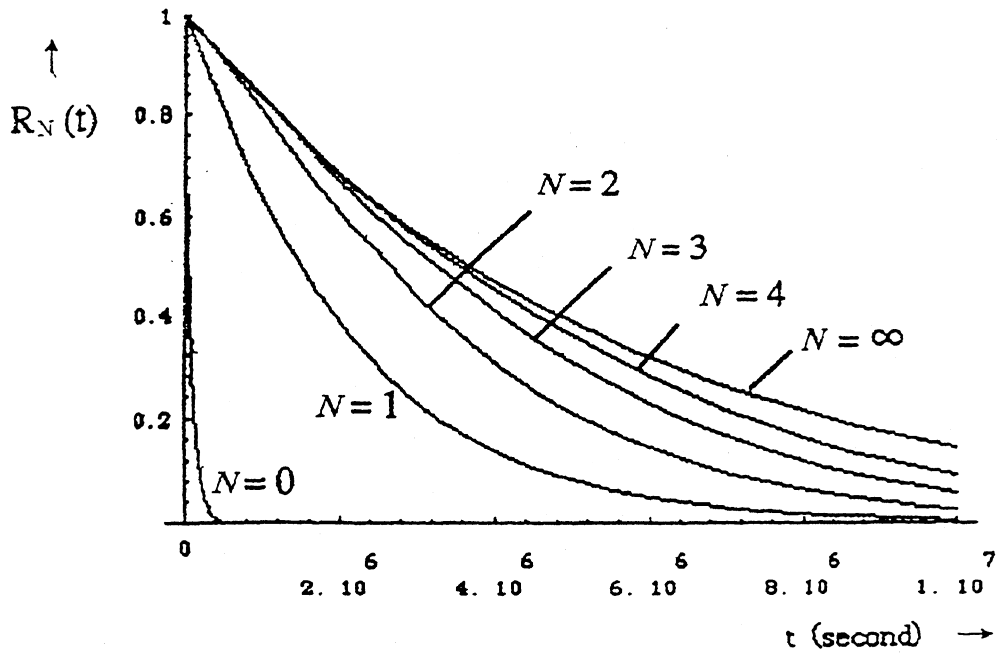


Figure 3.3: Reliability $R_N(t)$ when p=0.99 and θ =0.8.

Table 3.1: Optimal number N^* to minimize $C(N)$ when $1/\lambda = 1$ day, $p = 0.8$, $\theta = 0.8$.

$1/\alpha$	c_2/c_1						
(day)	10	10^2	10^3	10^4	10^5	10^6	10^7
30	1	2	4	5	6	7	8
60	1	2	3	4	5	5	6
90	1	2	3	3	4	5	6
180	1	2	2	3	3	4	5
365	1	1	2	2	3	3	4

Table 3.2: Numerical values for $\ell(N^*)$ when $1/\lambda = 1$ day, $1/\alpha = 180$ days.

($\times 10^6$ (seconds))

p	θ	c_2/c_1						
		10	10^2	10^3	10^4	10^5	10^6	10^7
0.8	0.8	0.423	0.430	0.430	0.430	0.430	0.430	0.430
	0.9	0.423	0.431	0.431	0.431	0.431	0.431	0.431
	0.99	0.423	0.432	0.432	0.432	0.432	0.432	0.432
0.9	0.8	0.823	0.854	0.855	0.855	0.855	0.855	0.855
	0.9	0.823	0.858	0.860	0.860	0.860	0.860	0.860
	0.99	0.823	0.861	0.863	0.864	0.864	0.864	0.864
0.99	0.8	7.156	7.733	7.780	7.784	7.785	7.785	7.785
	0.9	7.353	8.103	8.181	8.189	8.190	8.190	8.190
	0.99	8.217	8.546	8.587	8.592	8.593	8.593	8.593

Appendix

3.1. Mass functions $Q_{i,j}(t)$ $(i = 1, 2, \cdots, N; j = 1, 2, \cdots, N, F)$

The mass functions $Q_{i,j}(t)$ from state i at time 0 to state j at time t are given by the following equations:

$$Q_{i,i}(t) = p\int_0^t e^{-\alpha u} dF(u) \quad (i = 1, 2, \cdots, N), \tag{A3.1}$$

$$Q_{i,F}(t) = \left[\sum_{j=1}^{\infty} Q_{i,i}^{(j-1)}(t)\right] * \left[(1-p)\int_0^t e^{-\alpha u} dF(u) + (1-\theta)\int_0^t (1-e^{-\alpha u}) dF(u) \right.$$
$$\left. + \int_0^t \frac{\alpha\theta}{\alpha-\beta}(e^{-\beta u} - e^{-\alpha u}) dF(u)\right] \quad (i = 1, 2, \cdots, N-1), \tag{A3.2}$$

$$Q_{i,i+1}(t) = \left[\sum_{j=1}^{\infty} Q_{i,i}^{(j-1)}(t)\right] * \left[\int_0^t \frac{\alpha\beta\theta}{\alpha-\beta}(e^{-\beta u} - e^{-\alpha u})(1-F(u)) du\right]$$
$$(i = 1, 2, \cdots, N-1), \tag{A3.3}$$

$$Q_{N,F}(t) = \left[\sum_{j=1}^{\infty} Q_{N,N}^{(j-1)}(t)\right] * \left[(1-p)\int_0^t e^{-\alpha u} dF(u) + \int_0^t (1-e^{-\alpha u}) dF(u)\right], \tag{A3.4}$$

where the asterisk mark denotes the Stieltjes convolution, $a^{(n)}(t) (n = 1, 2, \cdots)$ denotes the n-fold Stieltjes convolution of a distribution $a(t)$ with itself and $a^{(0)}(t) \equiv 1$ for $t \geq 0, 0$ for $t < 0$, i.e., $a^{(n)}(t) \equiv a^{(n-1)}(t) * a(t)$, $a(t) * b(t) \equiv \int_0^t b(t-u) da(u)$. For example, $Q_{N,F}(t)$ is the probability distribution that when the N-th WDP is monitoring the MPu, the system transits to faulty state because of either case where errors of the MPu occur and a WDP cannot detect them although a WDP is normal or where errors of the MPu occur when a WDP is abnormal.

3.2. Derivation of equation (3.10)

Substituting $f(s) = \lambda/(s+\lambda)$ in (3.6), and for simplicity of the equation, assuming $x \equiv \alpha + \lambda(1-p)$, $y \equiv \beta + \lambda$, we have

$$h_N(s) = \sum_{j=0}^{N-2}\left[\frac{\alpha\beta\theta}{(s+x)(s+y)}\right]^j\left[\frac{\lambda}{s+\lambda} - \frac{sp\lambda}{(s+x)(s+\lambda)} - \frac{\lambda\alpha\beta\theta}{(s+x)(s+y)(s+\lambda)}\right]$$

$$+\left[\frac{\lambda}{s+\lambda} - \frac{sp\lambda}{(s+x)(s+\lambda)}\right]\left[\frac{\alpha\beta\theta}{(s+x)(s+y)}\right]^{N-1}$$

$$= \frac{\lambda}{s+\lambda}\left\{\sum_{j=0}^{N-2}[\frac{\alpha\beta\theta}{(s+x)(s+y)}]^j[1 - \frac{sp}{s+x} - \frac{\alpha\beta\theta}{(s+x)(s+y)}]\right.$$

$$\left.+(1 - \frac{sp}{s+x})[\frac{\alpha\beta\theta}{(s+x)(s+y)}]^{N-1}\right\}$$

$$= \frac{\lambda}{s+\lambda}\left\{1 - \frac{sp}{s+x}\sum_{j=0}^{N-1}[\frac{\alpha\beta\theta}{(s+x)(s+y)}]^j\right\} \quad (N = 1, 2, \cdots). \tag{A3.5}$$

3.3. Derivation of $R_N(t)$

We can derive $R_N(t)(N = 0, 1, 2, \cdots)$ one by one by taking the LS inverse transform of $h_N(s)$ in (3.10).

(i) When $N = 0$, evidently, we have

$$h_0(s) = \frac{\lambda}{s+\lambda}. \tag{A3.6}$$

Thus,

$$R_0(t) \equiv 1 - H_0(t) = e^{-\lambda t}. \tag{A3.7}$$

(ii) When $N = 1$,

$$h_1(s) = h_0(s) - \frac{sp\lambda}{(s+x)(s+\lambda)}. \tag{A3.8}$$

Thus,

$$R_1(t) \equiv 1 - H_1(t) = R_0(t) + ve^{-xt} - ve^{-\lambda t}, \tag{A3.9}$$

where

$$v \equiv -\frac{p\lambda}{\alpha - \lambda p}, \tag{A3.10}$$

(iii) When $N = 2$,

$$h_2(s) = h_1(s) - \frac{sp\lambda\alpha\beta\theta}{(s+x)^2(s+y)(s+\lambda)}. \tag{A3.11}$$

Thus,

$$\begin{aligned} R_2(t) &\equiv 1 - H_2(t) \\ &= R_1(t) + v_1te^{-xt} + v_2e^{-xt} + v_3e^{-yt} + v_4e^{-\lambda t}, \end{aligned} \tag{A3.12}$$

where

$$v_1 \equiv -\frac{p\lambda\alpha\beta\theta}{y-\lambda}\left(\frac{1}{x-\lambda} + \frac{1}{y-x}\right), \tag{A3.13}$$

$$v_2 \equiv \frac{p\lambda\alpha\beta\theta}{y-\lambda}\left[\frac{1}{(x-\lambda)^2} + \frac{1}{(y-x)^2}\right], \tag{A3.14}$$

$$v_3 \equiv -\frac{p\lambda\alpha\beta\theta}{(y-x)^2(y-\lambda)}, \tag{A3.15}$$

$$v_4 \equiv \frac{p\lambda\alpha\beta\theta}{(x-\lambda)^2(y-\lambda)}. \tag{A3.16}$$

Similarly, when $N = 3, 4, \cdots$, we can compute $R_N(t)$ successively.

3.4. Derivation of $R_\infty(t)$

Taking $N \to \infty$ in (A3.5), we have

$$\begin{aligned} h_\infty(s) &\equiv \lim_{N\to\infty} h_N(s) \\ &= \frac{\lambda}{s+\lambda}\left\{1 - \frac{sp}{s+x}\sum_{j=0}^{\infty}[\frac{\alpha\beta\theta}{(s+x)(s+y)}]^j\right\} \\ &= \frac{\lambda}{s+\lambda}\left[1 - \frac{sp(s+y)}{(s+x)(s+y)-\alpha\beta\theta}\right]. \end{aligned} \tag{A3.17}$$

Thus, we can derive $R_\infty(t)$ by taking the LS inverse transform of $h_\infty(s)$:

$$\begin{aligned} R_\infty(t) &\equiv 1 - H_\infty(t) \\ &= R_0(t) + p\lambda\left[\frac{y-w_1}{(w_1-w_2)(w_1-\lambda)}e^{-w_1 t}\right. \\ &\quad \left. - \frac{y-w_2}{(w_1-w_2)(w_2-\lambda)}e^{-w_2 t} + \frac{y-\lambda}{(w_1-\lambda)(w_2-\lambda)}e^{-\lambda t}\right], \end{aligned} \tag{A3.18}$$

where

$$w_1 \equiv \frac{1}{2}[x + y + \sqrt{(x-y)^2 + 4\alpha\beta\theta}], \tag{A3.19}$$

$$w_2 \equiv \frac{1}{2}[x + y - \sqrt{(x-y)^2 + 4\alpha\beta\theta}]. \tag{A3.20}$$

Chapter 4

Optimal Number of Microprocessor Units with Watchdog Processor

This chapter considers a system with N microprocessor (μP) units, where each μP unit consists of μP and watchdog processor (WDP): When errors of a μP have occurred, a WDP detects them and resets a μP to an initial state. The reset number is checked at constant time T. If more than K resets have been made at time T, a μP becomes permanent fault and one of other μP units in standby begins to operate. The mean time and the expected cost until system failure are derived. An optimal number N^* which minimizes the expected cost is discussed.

4.1 Introduction

Chapter 3 has considered the system where a main processor has several watchdog processors (WDPs), and has shown that it is effective to have at least one WDP. However, many microprocessor (μP) units which consist of μP and WDP have been recently used in actual fields. This chapter considers the following system with N μP units to improve its reliability by redundancy: Each μP unit consists of μP and WDP. When errors of a μP have occurred, a WDP detects them with a certain probability and resets a μP to an initial state. If a WDP cannot detect errors and the reset dose not succeed, the system fails. The successful numbers of resets are checked at constant time T. If more than K resets have been made at time T, it is judged that a μP is in permanent fault, and one of other μP units in standby begins to operate. The above procedures are repeated until all of N μP units have become fault.

For the above model, we derive the mean time until system failure, using the theory of Markov renewal processes [Osaki92]. Further, introducing the cost of a μP, we discuss analytically the problem to obtain how many number of μP units is optimal. Finally, numerical examples are given when failure times of a μP are exponential.

4.2 Model and Analysis

The system has N μP units, where each unit consists of μP and WDP shown in Figure 4.1. We assume that:

(1) Errors of a μP due to hardware errors and mistakes of memory access or control occur at a non-homogeneous Poisson process with an intensity function $\lambda(t)$ and a mean-value function $\Lambda(t)$, i.e., $\Lambda(t) \equiv \int_0^t \lambda(u)du$.

(2) A WDP can detect errors of a μP with probability $p(0 < p \leq 1)$ and resets a μP to an initial state. This probability p is called *coverage of a WDP*. A WDP works independently of a μP and does not fail.

(3) If more than K resets have occurred at time T where T is previously specified, we regard that a μP is in faulty state, and switches over to one of other μP units in standby automatically with probability $\theta(0 < \theta \leq 1)$. Any switching times are neglected. On the other hand, if less than K resets have occurred at time T, a μP finishes one processing and returns to an initial state.

(4) If a WDP cannot detect errors of a μP or if it cannot be switched over from a μP with fault to one of standby units, the system becomes failure.

From assumption **(1)**, the probability that the j-th number of errors have exactly occurred during (0, t] is given by $P_j(t) \equiv \{[\Lambda(t)]^j/j!\}e^{-\Lambda(t)}(j = 0, 1, 2, \cdots)$. Under the above assumptions, we define the following states of the system:

State j**:** The j-th μP unit begins to execute one processing $(j = 1, 2, \cdots, N)$.

State F**:** System failure occurs.

The system states defined above form a Markov renewal process where state F is an absorbing state. Transition diagram between system states is shown in Figure 4.2.

Let $Q_{ij}(t)(i = 1, 2, \cdots, N; j = 1, 2, \cdots, N, F)$ be one-step transition probabilities of a Markov renewal process. Then, the mass functions $Q_{i,j}(t)$ from state i at time 0 to state j at time t are:

$$
\begin{aligned}
Q_{i,i}(t) &= \sum_{j=0}^{K-1} \int_0^t p^j P_j(u) dA(u) \quad (i = 1, 2, \cdots, N), && (4.1) \\
Q_{i,i+1}(t) &= \sum_{n=0}^{\infty} [Q_{i,i}^{(n)}(t)] * \theta \sum_{j=K}^{\infty} \int_0^t p^j P_j(u) dA(u) \quad (i = 1, 2, \cdots, N-1), && (4.2) \\
Q_{i,F}(t) &= \sum_{n=0}^{\infty} [Q_{i,i}^{(n)}(t)] * \Big[(1-\theta) \sum_{j=K}^{\infty} \int_0^t p^j P_j(u) dA(u) \\
&\quad + \sum_{j=0}^{\infty} \int_0^t \overline{A}(u) p^j (1-p) P_j(u) \lambda(u) du\Big] \quad (i = 1, 2, \cdots, N-1), && (4.3) \\
Q_{N,F}(t) &= \sum_{n=0}^{\infty} [Q_{i,i}^{(n)}(t)] * \Big[\sum_{j=K}^{\infty} \int_0^t p^j P_j(u) dA(u) \\
&\quad + \sum_{j=0}^{\infty} \int_0^t \overline{A}(u) p^j (1-p) P_j(u) \lambda(u) du\Big], && (4.4)
\end{aligned}
$$

where the asterisk mark denotes the Stieltjes convolution, $a^{(n)}(t)$ denotes the n-fold Stieltjes convolution of a distribution $a(t)$ with itself and $a^{(0)}(t) \equiv 1$ for $t \geq 0, 0$ for $t < 0$, i.e., $a^{(n)}(t) \equiv a^{(n-1)}(t) * a(t), a(t) * b(t) \equiv \int_0^t b(t-u) da(u)$, and

$$
A(t) \equiv \begin{cases} 1: & t \geq T, \\ 0: & t < T, \end{cases} \tag{4.5}
$$

is the degenerate distribution placing unit mass at T, $\overline{A}(t) \equiv 1 - A(t)$. For example, $Q_{i,F}(t)$ is the probability distribution that when the i-th μP unit is operating, the system transits to failure state because of either case where the switch-over to one of standby units fails when more than K resets have occurred at time T, or where a WDP cannot detect errors of a μP until time T.

Let $\phi(s)$ be the Laplace-Stieltjes (LS) transform of any function $\Phi(t)$. Taking the LS transforms of (4.1) $\sim$ (4.4), we have

$$
\begin{aligned}
q_{i,i}(s) &= \sum_{j=0}^{K-1} p^j e^{-sT} P_j(T) \quad (i = 1, 2, \cdots, N), && (4.6)\\
q_{i,i+1}(s) &= \sum_{n=1}^{\infty}\Big[\sum_{j=0}^{K-1} p^j e^{-sT} P_j(T)\Big]^{n-1} \times \Big[\theta \sum_{j=K}^{\infty} p^j e^{-sT} P_j(T)\Big] \\
&\quad (i = 1, 2, \cdots, N-1), && (4.7)\\
q_{i,F}(s) &= \sum_{n=1}^{\infty}\Big[\sum_{j=0}^{K-1} p^j e^{-sT} P_j(T)\Big]^{n-1} \\
&\quad \times\Big[(1-\theta)\sum_{j=K}^{\infty} p^j e^{-sT} P_j(T) + \sum_{j=0}^{\infty}\int_0^T e^{-st} p^j (1-p) P_j(t)\lambda(t)dt\Big] \\
&\quad (i = 1, 2, \cdots, N-1), && (4.8)\\
q_{N,F}(s) &= \sum_{n=1}^{\infty}\Big[\sum_{j=0}^{K-1} p^j e^{-sT} P_j(T)\Big]^{n-1} \\
&\quad \times\Big[\sum_{j=K}^{\infty} p^j e^{-sT} P_j(T) + \sum_{j=0}^{\infty}\int_0^T e^{-st} p^j (1-p) P_j(t)\lambda(t)dt\Big]. && (4.9)
\end{aligned}
$$

We derive the mean time $\ell(N)$ from the beginning of system operation to system failure. Let $H_N(t)$ be the first-passage time distribution from state 1 to state F. Then, we have

$$
H_N(t) = Q_{1,F}(t) + Q_{1,2}(t) * Q_{2,F}(t) + \cdots + Q_{1,2}(t) * \cdots * Q_{N-1,N}(t) * Q_{N,F}(t). \quad (4.10)
$$

Taking the LS transforms on both sides of (4.10) and arranging them, we have

$$
\begin{aligned}
h_N(s) \;=\; & \sum_{i=1}^{N-2}\Big[\frac{\theta\sum_{j=K}^{\infty}p^j e^{-sT}P_j(T)}{1-\sum_{j=0}^{K-1}p^j e^{-sT}P_j(T)}\Big]^i \\
& \times\frac{(1-\theta)\sum_{j=K}^{\infty}p^j e^{-sT}P_j(T)+\sum_{j=0}^{\infty}\int_0^T e^{-st}p^j(1-p)P_j(t)\lambda(t)dt}{1-\sum_{j=0}^{K-1}p^j e^{-sT}P_j(T)} \\
& +\Big[\frac{\theta\sum_{j=K}^{\infty}p^j e^{-sT}P_j(T)}{1-\sum_{j=0}^{K-1}p^j e^{-sT}P_j(T)}\Big]^{N-1} \\
& \times\frac{\sum_{j=K}^{\infty}p^j e^{-sT}P_j(T)+\sum_{j=0}^{\infty}\int_0^T e^{-st}p^j(1-p)P_j(t)\lambda(t)dt}{1-\sum_{j=0}^{K-1}p^j e^{-sT}P_j(T)} \quad (N=1,2,\cdots),
\end{aligned}
\tag{4.11}
$$

where $\sum_{i=1}^{-1}\equiv 0$. Hence, the mean time $\ell(N)$ to system failure is

$$
\begin{aligned}
\ell(N) &\equiv \int_0^\infty t\,dH_N(t) \\
&= \frac{1-A^N}{1-A}B \quad (N=1,2,\cdots),
\end{aligned}
\tag{4.12}
$$

where

$$
A \equiv \frac{\theta\sum_{j=K}^{\infty}p^jP_j(T)}{1-\sum_{j=0}^{K-1}p^jP_j(T)}, \tag{4.13}
$$

$$
B \equiv \frac{\int_0^T e^{-(1-p)\Lambda(t)}dt}{1-\sum_{j=0}^{K-1}p^jP_j(T)}. \tag{4.14}
$$

It can be easily seen that $A = q_{i,i+1}(0)$ in (4.7) and $\ell(N)$ is increasing from B to $B/(1-A)$.

4.3 Optimal Policy

Let c_1 be the cost for a μP unit and c_2 be the cost for system failure. Then, the expected cost $C(N)$ per unit of time of the system with N μP units is given by

$$\begin{aligned} C(N) &\equiv \frac{Nc_1 + c_2}{\ell(N)} \\ &= \frac{Nc_1 + c_2}{\dfrac{1-A^N}{1-A}B} \qquad (N = 1, 2, \cdots). \end{aligned} \tag{4.15}$$

We seek an optimal number N^* which minimizes $C(N)$ in (4.15). From the inequality $C(N+1) - C(N) \geq 0$, we have

$$\frac{1-A^N}{A^N(1-A)} - N \geq \frac{c_2}{c_1}. \tag{4.16}$$

Denoting the left side of (4.16) by $L(N)$, we have

$$L(N) - L(N-1) = \frac{1+A^N}{A^N} > 0, \tag{4.17}$$

and

$$L(1) = \frac{1-A}{A}, \tag{4.18}$$

$$L(\infty) \equiv \lim_{N\to\infty} L(N) = \infty. \tag{4.19}$$

Hence, $L(N)$ is strictly increasing in N from $(1-A)/A$ to ∞. Thus, we have the following optimal policy:

(i) If $(1-A)/A < c_2/c_1$, then there exists a finite and unique minimum N^* which satisfies (4.16).

(ii) If $(1-A)/A \geq c_2/c_1$, then $N^* = 1$. In this case, it is evident that

$$\frac{1-A}{A} = \frac{1}{\theta}\Bigg[\frac{1-\sum_{j=0}^{\infty} p^j P_j(T)}{\sum_{j=K}^{\infty} p^j P_j(T)} + 1\Bigg] - 1,$$

which is an increasing function of K. Thus, if K increases then the case of $N^* = 1$ increases. We can compute a minimum value K which satisfies $(1-A)/A \geq c_2/c_1$ for the case where $N^* = 1$.

4.4 Numerical Examples

We compute numerically the mean time $\ell(N)$ and the expected cost $C(N)$ when errors of a μP occur at a Poisson process with constant rate λ. Suppose that the coverage of a WDP is $p = 0.8 \sim 0.99$, the probability that the switch-over from a μP to other units in standby succeeds is $\theta = 0.9 \sim 1.0$, and the cost rate of system failure to a μP is $c_2/c_1 = 10^2 \sim 10^4$. Further, the interval time T of checks per the mean time $1/\lambda$ of error occurrences of a μP is $\lambda T = 10^{-4} \sim 10^{-1}$ and the upper limit number of resets is $K = 2 \sim 4$.

Table 4.1 gives the optimal number N^* which minimizes the expected cost $C(N)$. This indicates that N^*'s decrease with K, however, increase with p, λT and c_2/c_1. For example, when $\theta = 0.9$, $p = 0.9$, $\lambda T = 10^{-2}$, $K = 3$ and $c_2/c_1 = 10^4$, the optimal number is $N^* = 2$. This also indicates that N^*'s depend little on θ and are almost 1 for $K \geq 4$. Therefore, we can conclude that the system is enough to have only one unit when the reset number K takes ordinary values from 4 to 8.

Figure 4.3 draws $\ell(1)$ for $1/\lambda$ and $p = 0.8, 0.9, 0.99$ when $T = 1$ second and $K = 4$. This indicates that $\ell(1)$ increases noticeably with p. That is, to develop the reliability of the system, we should improve the coverage of a WDP.

Moreover, we compute a minimum value K for the case where $N^* = 1$ which satisfies $(1-A)/A \geq c_2/c_1$ in Table 4.2. This indicates that these values increase with p and

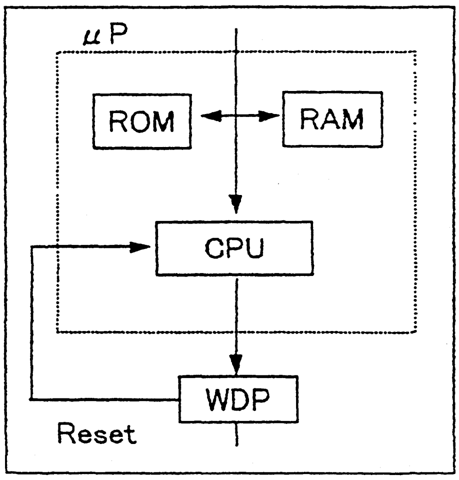
λT.

4.5 Conclusions

It would be very important to evaluate and improve the reliability of systems with μP. This chapter has considered a redundant system with N μP units to improve the reliability. Under the assumption that a μP is in faulty state if more than K resets have occurred at time T, we have derived the mean time and the expected cost until system failure. Further, we have discussed an optimal number N^* which minimizes the expected cost.

From the numerical examples, it has been shown that the optimal number is almost 1 for $K \geq 4$, and hence, the system is enough to have only one unit. Further, we have understood that the probability of coverage of a WDP gives a great influence on the improvement of the system.

A μ P unit



The system with N μ P units

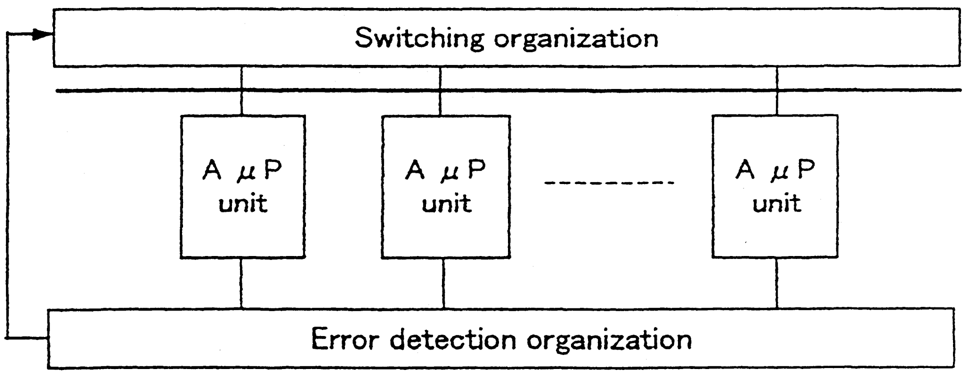


Figure 4.1 : Outline of the model.

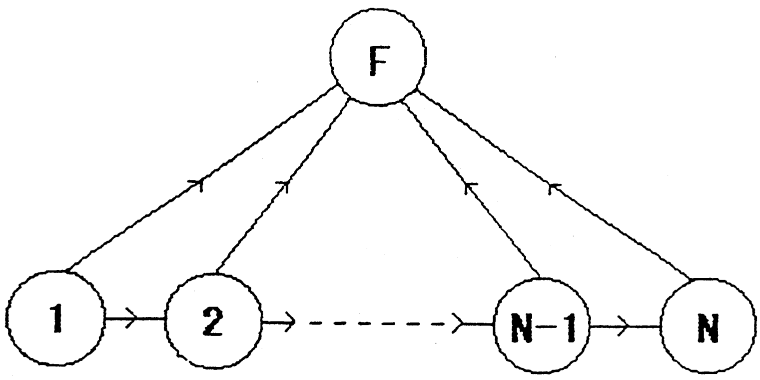


Figure 4.2: Transition diagram between system states.

Table 4.1: Optimal number N^* to minimize $C(N)$.

θ	p	K	$\lambda T = 10^{-4}$			$\lambda T = 10^{-3}$			$\lambda T = 10^{-2}$			$\lambda T = 10^{-1}$		
			c_2/c_1			c_2/c_1			c_2/c_1			c_2/c_1		
			10^2	10^3	10^4	10^2	10^3	10^4	10^2	10^3	10^4	10^2	10^3	10^4
0.9	0.8	2	1	1	2	1	2	2	2	2	3	3	4	5
		3	1	1	1	1	1	1	1	1	1	1	2	2
		4	1	1	1	1	1	1	1	1	1	1	1	1
	0.9	2	1	1	2	1	2	2	2	3	3	4	5	7
		3	1	1	1	1	1	1	1	1	2	2	2	3
		4	1	1	1	1	1	1	1	1	1	1	1	2
1.0	0.8	2	1	1	2	1	2	2	2	2	3	3	4	5
		3	1	1	1	1	1	1	1	1	1	1	2	2
		4	1	1	1	1	1	1	1	1	1	1	1	1
	0.9	2	1	1	2	1	2	2	2	3	3	4	6	7
		3	1	1	1	1	1	1	1	1	2	2	2	3
		4	1	1	1	1	1	1	1	1	1	1	1	2

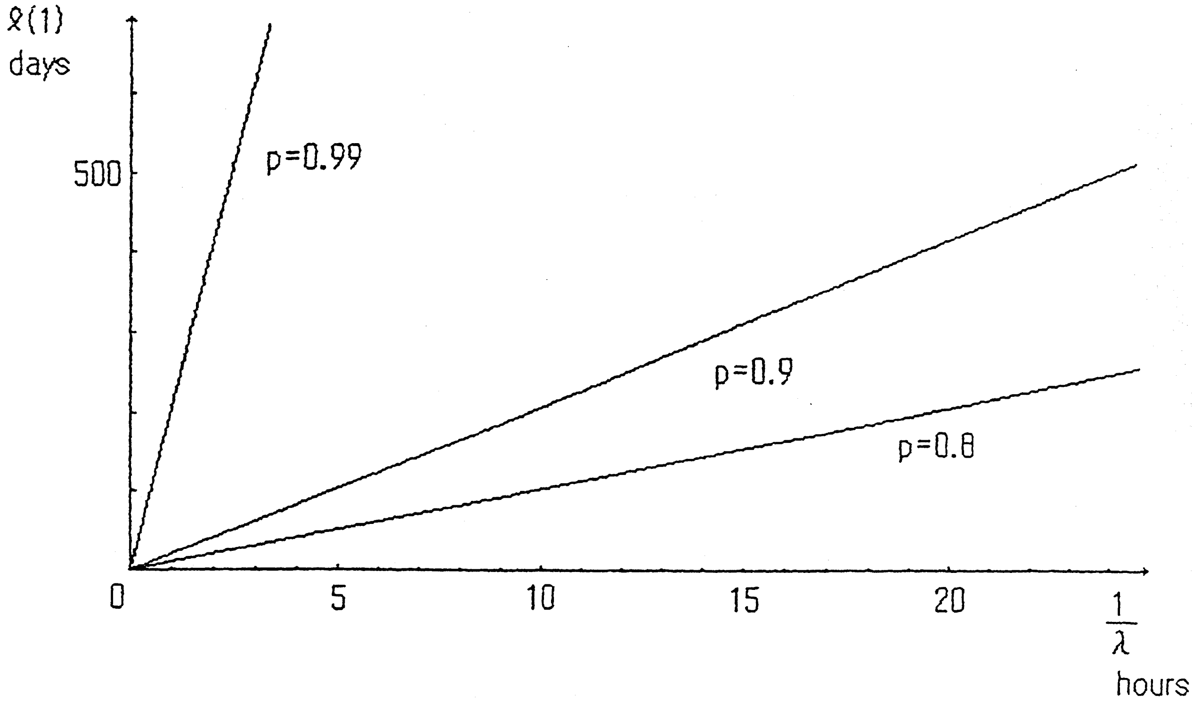


Figure 4.3: Mean time $\ell(1)$ to system failure when N=1, T=1 second, K=4 and p=0.8, 0.9, 0.99.

Table 4.2: Minimum value K for the case where $N^* = 1$.

θ	p	$\lambda T = 10^{-4}$			$\lambda T = 10^{-3}$			$\lambda T = 10^{-2}$			$\lambda T = 10^{-1}$		
		c_2/c_1			c_2/c_1			c_2/c_1			c_2/c_1		
		10^2	10^3	10^4	10^2	10^3	10^4	10^2	10^3	10^4	10^2	10^3	10^4
0.9	0.8	2	2	3	2	3	3	3	3	3	3	4	4
	0.9	2	2	3	2	3	3	3	3	4	4	4	5
1.0	0.8	2	2	3	2	3	3	3	3	3	3	4	4
	0.9	2	2	3	2	3	3	3	3	4	4	4	5

Chapter 5

Reliability Evaluations of a Microprocessor System with Limit Processing Time

This chapter considers reliability problems of a system with N microprocessor (μP) units where each μP unit consists of μP and watchdog processor: If the operating unit cannot finish one processing by errors until a limit time, it changes to one of standby units. The mean time and the expected number of processings until system failure are obtained. Using these results, the cost effectiveness is derived and an optimal number of μP units which minimizes it is analytically discussed. Finally, numerical examples are given under suitable conditions.

5.1 Introduction

A large number of microprocessors (μPs) have been widely used in many practical fields. A watchdog processor (WDP) is the most convenient coprocessor to monitor the behavior of a μP since it is simple and low-priced.

Yasui, et al. (1994) [YNH94] considered a μP system with watchdog timer (WDT) which is simplified a WDP function. They also showed that it is effective to have a WDT for the system which demands a high reliability.

This chapter considers the reliability problems of a system with limit processing time: The system consists of N μP units where each μP has a WDP. When errors of a μP have occurred, a WDP detects them and resets a μP to an initial state. Otherwise, the system goes to failure. If the operating unit cannot finish one processing until a limit time T, a μP becomes faulty and its unit changes to one of the standby units. The above procedures are repeated until all of N units have become faulty.

The mean time and the expected number of processings until system failure are obtained. Using these results, the cost effectiveness is derived and an optimal number of μP units which minimizes it is analytically discussed. Finally, numerical examples are given.

5.2 Model and Analysis

The system has N μP units where each unit consists of μP and WDP shown in Figure 5.1. We regard that a μP becomes faulty if it does not finish one processing until a limit time T.

(1) A μP repeats one time of processing which takes the total time of a main processing and an initial processing for preparation to execute a main processing. Its initial processing and main processing need the respective times according to exponential distributions $(1 - e^{-\alpha t})$ and $(1 - e^{-\beta t})$.

(2) Errors of a μP occur according to a general distribution $F(t)$ with finite mean $1/\lambda$.

(3) A WDP can detect errors of a μP with probability $p(0 < p \leq 1)$ and resets a μP to an initial state of a main processing. This probability p is called *coverage of a WDP*. A WDP works independently of a μP and does not fail.

(4) If the operating unit cannot finish one processing by errors until a limit time T, it changes to one of the standby units. The probability that the switch-over from a μP unit to other units in standby succeed is θ $(0 < \theta \leq 1)$, and its switch-over time is constant v.

(5) If a WDP cannot detect errors of a μP, if it cannot be switched over from a faulty μP to one of the standby units or if errors of a μP occur before a μP finishes an initial processing, the system becomes failure. Besides, if the N-th operating unit cannot finish one processing until a limit time T, the system also becomes failure.

Under the above assumptions, we define the following states of the system:

State i**:** The i-th μP unit begins to execute one processing $(i = 1, 2, \cdots, N)$.

State F**:** System failure occurs.

The system states defined above a Markov renewal process [Osaki92] where state F is an absorbing state. Transition diagram between system states is shown in Figure 5.2. We define the distribution $U(t)$ of a limit time T and the distribution $V(t)$ of the processing time of switching as the following functions:

$$U(t) \equiv \begin{cases} 1: & t \geq T, \\ 0: & t < T, \end{cases} \tag{5.1}$$

$$V(t) \equiv \begin{cases} 1: & t \geq v, \\ 0: & t < v. \end{cases} \tag{5.2}$$

Let $Q_{i,j}(t)(i = 1, 2, \cdots, N; j = 1, 2, \cdots, N, F)$ be one-step transition probabilities of a Markov renewal process and $\phi(s)$ be the Laplace-Stieltjes (LS) transform of any function $\Phi(t)$, i.e., $\phi(s) \equiv \int_0^\infty e^{-st} d\Phi(t)$ for $Re(s) > 0$. Further, we put that

$$h_T(s) \equiv \int_0^T e^{-(s+\beta)t} dF(t). \tag{5.3}$$

Then, from Appendix 5.1,

$$q_{i,i}(s) = \frac{\alpha\beta[1-f(s+\alpha)]}{(s+\alpha)(s+\beta)} \cdot \frac{1-\overline{F}(T)e^{-(s+\beta)T}-h_T(s)}{1-ph_T(s)} \qquad (i=1,2,\cdots,N), \tag{5.4}$$

$$q_{i,i+1}(s) = \frac{\alpha\theta(s+\beta)[1-f(s+\alpha)]e^{-\beta T}e^{-s(T+v)}\overline{F}(T)}{\left\{\begin{array}{l}(s+\alpha)(s+\beta)[1-ph_T(s)] \\ -\alpha\beta[1-f(s+\alpha)][1-\overline{F}(T)e^{-(s+\beta)T}-h_T(s)]\end{array}\right\}} \quad (i=1,2,\cdots,N-1), \tag{5.5}$$

$$q_{i,F}(s) = \frac{\left\{\begin{array}{l}(s+\beta)[1-ph_T(s)][\alpha+sf(s+\alpha)] \\ -\alpha(s+\beta)[1-f(s+\alpha)] \\ \times[1-h_T(s)-(1-\theta)e^{-\beta T}e^{-s(T+v)}\overline{F}(T)]\end{array}\right\}}{\left\{\begin{array}{l}(s+\alpha)(s+\beta)[1-ph_T(s)] \\ -\alpha\beta[1-f(s+\alpha)][1-\overline{F}(T)e^{-(s+\beta)T}-h_T(s)]\end{array}\right\}} \quad (i=1,2,\cdots,N-1), \tag{5.6}$$

$$q_{N,F}(s) = \frac{\left\{\begin{array}{l}(s+\alpha)(s+\beta)[1-ph_T(s)]f(s+\alpha) \\ +\alpha(s+\beta)[1-f(s+\alpha)] \\ \times[(1-p)h_T(s)+e^{-(s+\beta)T}\overline{F}(T)]\end{array}\right\}}{\left\{\begin{array}{l}(s+\alpha)(s+\beta)[1-ph_T(s)] \\ -\alpha\beta[1-f(s+\alpha)][1-\overline{F}(T)e^{-(s+\beta)T}-h_T(s)]\end{array}\right\}}. \tag{5.7}$$

Note that $q_{i,j}(s)$ do not depend on i in (5.4) $\sim$ (5.6).

We derive the mean time $\ell(N)$ to system failure. Let $H_N(t)$ be the first-passage time distribution from state 1 to state F. Then, we have

$$H_N(t) = Q_{1,F}(t) + Q_{1,2}(t) * Q_{2,F}(t) + \cdots + Q_{1,2}(t) * \cdots * Q_{N-1,N}(t) * Q_{N,F}(t). \tag{5.8}$$

Hence, the mean time $\ell(N)$ to system failure is

$$\begin{aligned} \ell(N) &\equiv \int_0^\infty t dH_N(t) = \lim_{s\to 0}\frac{d}{ds}[-h_N(s)] \\ &= \frac{1}{1-q}[D(1-q^N) + E(1-q^{N-1})] \quad (N=1,2.\cdots), \end{aligned} \tag{5.9}$$

where

$$
\begin{aligned}
q &\equiv q_{i,i+1}(0) \\
&= \frac{\theta[1-f(\alpha)]\overline{F}(T)e^{-\beta T}}{1-ph_T(0)-[1-f(\alpha)][1-\overline{F}(T)e^{-\beta T}-h_T(0)]}, &(5.10)\\
D &\equiv \frac{\left\{\frac{1}{\alpha}[1-ph_T(0)]+\frac{1}{\beta}[1-h_T(0)-\overline{F}(T)e^{-\beta T}]\right\}[1-f(\alpha)]}{1-ph_T(0)-[1-f(\alpha)][1-h_T(0)-\overline{F}(T)e^{-\beta T}]}, &(5.11)\\
E &\equiv \frac{v\overline{F}(T)e^{-\beta T}[1-f(\alpha)]}{1-ph_T(0)-[1-f(\alpha)][1-h_T(0)-\overline{F}(T)e^{-\beta T}]}. &(5.12)
\end{aligned}
$$

Note that $0 < q < 1$, and for $N = 1, \infty$, we have, respectively,

$$
\begin{aligned}
\ell(1) &= D, &(5.13)\\
\ell(\infty) &= \frac{D+E}{1-q}. &(5.14)
\end{aligned}
$$

Next, we derive the expected number of processings to system failure. The expected number $M_i(t)$ of visits to state i until time t, when the system starts from state i at time 0, is given by the following renewal equation:

$$M_i(t) = Q_{i,i}(t) * [1 + M_i(t)] \quad (i = 1, 2, \cdots, N). \tag{5.15}$$

Thus, the LS transform $m(s)$ of the expected number $M(t)$ of processings until the system moves from state 1 at time 0 to state F is given by

$$
\begin{aligned}
m(s) &= m_1(s) + q_{1,2}(s)m_2(s) + \cdots + q_{1,2}(s)q_{2,3}(s)\cdots q_{N-1,N}(s)m_N(s) \\
&= \sum_{j=1}^{N} m_1(s)[q_{i,i+1}(s)]^{j-1}, &(5.16)
\end{aligned}
$$

where $m_1(s) \equiv m_i(s)$ $(i = 1, 2, \cdots, N)$. Therefore, from $q_{i,i+1}(0) = q$, we derive the expected number M of processings until system failure in the following equation:

$$M \equiv \lim_{t\to\infty} M(t) = \lim_{s\to 0} m(s)$$

$$
\begin{aligned}
&= \sum_{j=1}^{N} m_1(0) q^{j-1} \\
&= \frac{[1-f(\alpha)][1-\overline{F}(T)e^{-\beta T}-h_T(0)]}{\left\{\begin{array}{l} 1-ph_T(0)-[1-f(\alpha)][1-\overline{F}(T)e^{-\beta T}-h_T(0)] \\ -\theta e^{-\beta T}[1-f(\alpha)]\overline{F}(T) \end{array}\right\}} \\
&\quad \times \left[1-\left\{\frac{\theta e^{-\beta T}[1-f(\alpha)]\overline{F}(T)}{1-ph_T(0)-[1-f(\alpha)][1-\overline{F}(T)e^{-\beta T}-h_T(0)]}\right\}^{N}\right]. \qquad (5.17)
\end{aligned}
$$

5.3 Optimal Policy

Generally, the expected cost would be mutually exclusive against the effectiveness. We discuss an optimal policy by introducing the concept of cost effectiveness: Let c_1 be the acquisition cost for a μP unit and c_2 be the cost for system failure. Then, we assume that the expected cost per unit of time of the system with N μP units is $\tilde{C}(N) \equiv (Nc_1 + c_2)/\ell(N)$, and the effectiveness which is the expected number of processings per unit of time is $M/\ell(N)$. Then, we define the cost / the effectiveness as the following equation:

$$
C(N) \equiv \frac{\tilde{C}(N)}{\frac{M}{\ell(N)}} = \frac{Nc_1 + c_2}{M}. \qquad (5.18)
$$

That is, $C(N)$ denotes the expected cost per one time of processing. From equation (5.17), we have

$$
C(N) = \frac{Nc_1 + c_2}{\sum_{j=1}^{N} m_1(0) q^{j-1}}. \qquad (5.19)
$$

Note that

$$
A_j \equiv m_1(0) q^{j-1}, \qquad (5.20)
$$

is strictly decreasing in j since $0 < q < 1$, and hence, $\lim_{j\to\infty} A_j = 0$.

We seek an optimal number N^* which minimizes $C(N)$ in (5.19). From the inequality $C(N+1) - C(N) \geq 0$, we have

$$\frac{1}{A_{N+1}} \sum_{j=1}^{N} A_j - N \geq \frac{c_2}{c_1}. \tag{5.21}$$

Denoting the left side of (5.21) by $L(N)$, we have

$$\begin{aligned}
L(N) - L(N-1) &= \sum_{j=1}^{N} A_j \left(\frac{1}{A_{N+1}} - \frac{1}{A_N} \right) > 0, & (5.22)\\
L(1) &= \frac{A_1}{A_2} - 1 = \frac{1}{q} - 1 > 0, & (5.23)\\
L(\infty) &= \lim_{N \to \infty} \left(\frac{\sum_{j=1}^{N} A_j}{A_{N+1}} - N \right) \\
&\geq \lim_{N \to \infty} \frac{A_1}{A_{N+1}} - 1 = \infty. & (5.24)
\end{aligned}$$

Hence, $L(N)$ is strictly increasing in N from $L(1)$ to ∞.

Thus, we have the following optimal policy:

(i) If $L(1) \geq c_2/c_1$, i.e., $q \leq c_1/(c_1 + c_2)$ then $N^* = 1$.

(ii) If $L(1) < c_2/c_1$, i.e., $q > c_1/(c_1+c_2)$ then there exists a finite and unique minimum $N^*(> 1)$ which satisfies (5.21).

5.4 Numerical Examples

We compute numerically the optimal number N^* which minimizes the cost / the effectiveness $C(N)$.

Suppose that errors of a μP occur according to an exponential distribution $F(t) = 1 - e^{-\lambda t}$ and the mean main processing time $1/\beta$ of μP is a unit time of the system. Further, suppose that the mean time to error occurrences is $(1/\lambda)/(1/\beta) = 3600 \sim$

3600×24 (when $1/\beta = 1$ second, $1/\lambda$ corresponds to $1 \sim 24$ hours), the mean initial processing time is $(1/\alpha)/(1/\beta) = 1$, the mean processing time of the switching of the μP unit is $v/(1/\beta) = 1/(30 \times 10^4)$. Moreover, the probability that the switching of μP unit succeeds is $\theta = 0.8 \sim 0.99$, the coverage of a WDP is $p = 0.8 \sim 0.99$, the acquisition cost c_1 for a μP unit is a unit of cost and the cost rate of system failure for a μP unit is $c_2/c_1 = 10 \sim 10^3$.

Table 5.1 gives the optimal number N^* which minimizes the expected cost $C(N)$ when a limit processing time T of μP is $10 \sim 20$ times of the main processing time $1/\beta$ of μP, i.e., $T/(1/\beta) = \beta T = 10 \sim 20$. This indicates that N^* decreases with βT, however, increases with $1/\lambda$, p, θ and c_2/c_1. For example, when $(1/\lambda)/(1/\beta) = 3600 \times 24$, $p = 0.9$, $\theta = 0.9$, $\beta T = 15$ and $c_2/c_1 = 10^2$, the optimal number of μP units is $N^* = 2$. This also indicates that N^* depends on $1/\lambda$, p and θ when βT takes small values, however, when $\beta T \geq 15$, N^* depends little on them and N^* is almost $1 \sim 2$.

Next, Figure 5.3 draws $C(N)$ for N and gives the optimal number N^* when $(1/\lambda)/(1/\beta) = 3600,\ 3600 \times 24$, $p = 0.8$, $\theta = 0.8$, $\beta T = 10$ and $c_2/c_1 = 10$. This indicates that $C(N)$ decreases noticeably with $1/\lambda$. We can consider that N^* increases with $1/\lambda$ in Table 5.1 so that the processing number of μP within a limit processing time βT increases and the expected cost decreases remarkably. That is, from Figure 5.3, as the μP unit becomes advanced, it seems that the optimal number N^* becomes large so as to decrease the expected cost for the effectiveness.

5.5 Conclusions

We have considered the reliability problems of a system with N μP units. Under the assumption that a μP is in faulty state if it does not finish one processing until a limit time T, we have derived the mean time, the expected number of processings until system failure by considering the mean time to error occurrences of a μP, the coverage of a WDP and so on. Further, introducing the concept of cost effectiveness, we have discussed an optimal number which minimizes the expected cost for the effectiveness.

From the numerical examples, it has been shown that the optimal number N^* which minimizes the cost / the effectiveness decreases with βT, however, increases with $1/\lambda$, p, θ, and c_2/c_1, and N^* depends little on them and N^* is almost $1 \sim 2$ when $\beta T \geq 15$. Further, an interesting consequence has been obtained that when βT is small comparatively, as the μP unit becomes advanced, the expected cost per unit of processing decreases, and oppositely, the optimal number N^* increases.

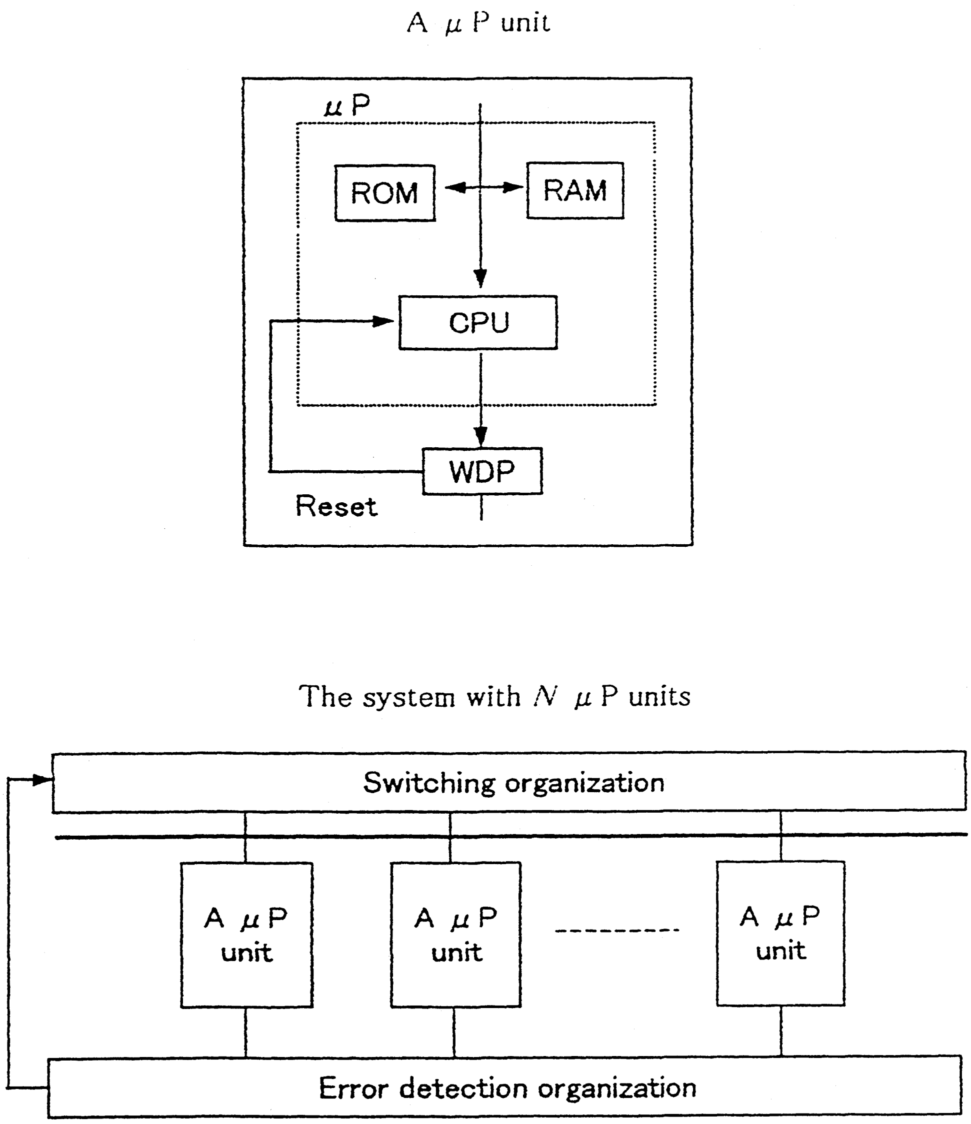


Figure 5.1: Outline of the model.

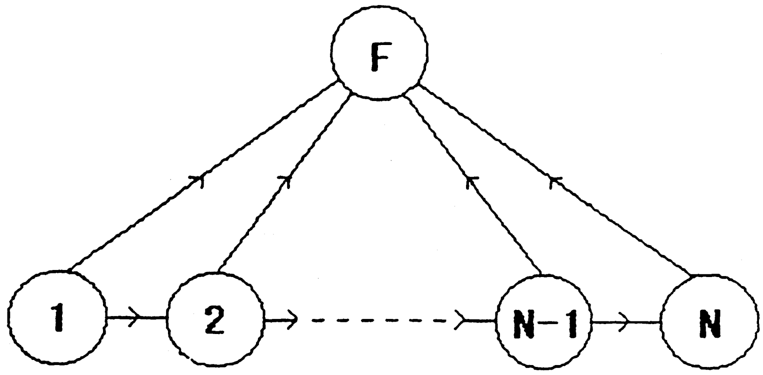


Figure 5.2: Transition diagram between system states.

Table 5.1: Optimal number N^* to minimize $C(N)$.

$(1/\lambda)/(1/\beta)$	p	θ	βT = 10, c_2/c_1 = 10	10^2	10^3	βT = 15, c_2/c_1 = 10	10^2	10^3	βT = 20, c_2/c_1 = 10	10^2	10^3
3600	0.8	0.8	2	4	6	1	1	2	1	1	1
		0.9	2	4	6	1	1	2	1	1	1
		0.99	3	5	7	1	1	2	1	1	1
	0.9	0.8	3	5	7	1	1	2	1	1	1
		0.9	3	5	8	1	1	2	1	1	1
		0.99	3	6	8	1	1	2	1	1	1
	0.99	0.8	3	6	9	1	1	2	1	1	1
		0.9	4	7	10	1	1	2	1	1	1
		0.99	4	8	12	1	1	2	1	1	1
3600 × 24	0.8	0.8	4	8	13	1	2	2	1	1	1
		0.9	5	10	16	1	2	2	1	1	1
		0.99	6	13	20	1	2	2	1	1	1
	0.9	0.8	4	8	13	1	2	2	1	1	1
		0.9	5	10	17	1	2	2	1	1	1
		0.99	6	13	22	1	2	2	1	1	1
	0.99	0.8	4	9	14	1	2	2	1	1	1
		0.9	5	11	17	1	2	2	1	1	1
		0.99	7	14	23	1	2	2	1	1	1

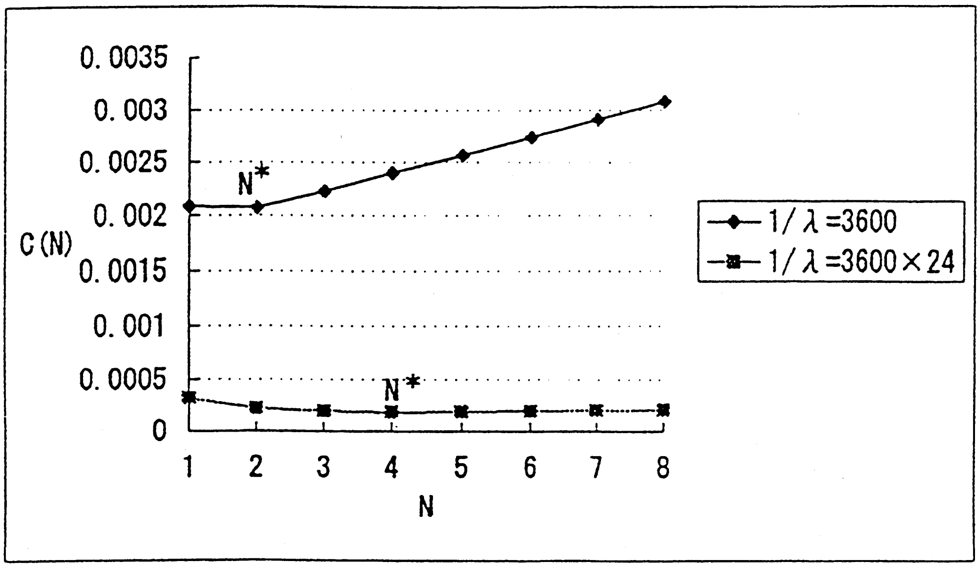


Figure 5.3: Cost effectiveness C(N) for N and N^* when $(1/\lambda)/(1/\beta)=3600$, 3600×24, $p=0.8$, $\theta=0.8$, $\beta T=10$ and $c_2/c_1=10$.

Appendix

5.1. Mass functions $Q_{i,j}(t)(i = 1, 2, \cdots, N; j = 1, 2, \cdots, N, F)$

The mass functions $Q_{i,j}(t)$ from state i at time 0 to state j at time t are given in the following equations:

$$Q_{i,i}(t) = \Big[\int_0^t \overline{F}(u)dA(u)\Big] * \Big\{\sum_{k=1}^{\infty}\Big[p\int_0^t \overline{B}(u)\overline{U}(u)dF(u)\Big]^{(k-1)} * \int_0^t \overline{F}(u)\overline{U}(u)dB(u)\Big\}$$
$$(i = 1, 2, \cdots, N), \qquad (A5.1)$$

$$Q_{i,i+1}(t) = \sum_{j=1}^{\infty}\Big[Q_{i,i}(t)\Big]^{(j-1)} * \Big[\int_0^t \overline{F}(u)dA(u)\Big] * \Big\{\sum_{k=1}^{\infty}\Big[p\int_0^t \overline{B}(u)\overline{U}(u)dF(u)\Big]^{(k-1)}$$
$$*\Big[\int_0^t \overline{F}(u)\overline{B}(u)dU(u)\Big]\Big\}*\Big[\theta V(t)\Big] \quad (i = 1, 2, \cdots, N-1), \qquad (A5.2)$$

$$Q_{i,F}(t) = \sum_{j=1}^{\infty}\Big[Q_{i,i}(t)\Big]^{(j-1)} * \Big[\int_0^t \overline{A}(u)dF(u)\Big]$$
$$+\sum_{j=1}^{\infty}\Big[Q_{i,i}(t)\Big]^{(j-1)} * \Big[\int_0^t \overline{F}(u)dA(u)\Big] * \sum_{k=1}^{\infty}\Big[p\int_0^t \overline{B}(u)\overline{U}(u)dF(u)\Big]^{(k-1)}$$
$$*\Big\{(1-p)\int_0^t \overline{B}(u)\overline{U}(u)dF(u) + \Big[\int_0^t \overline{F}(u)\overline{B}(u)dU(u)\Big] * \Big[(1-\theta)V(t)\Big]\Big\}$$
$$(i = 1, 2, \cdots, N-1), \qquad (A5.3)$$

$$Q_{N.F}(t) = \sum_{j=1}^{\infty}\Big[Q_{N,N}(t)\Big]^{(j-1)} * \Big[\int_0^t \overline{A}(u)dF(u)\Big]$$
$$+\sum_{j=1}^{\infty}\Big[Q_{N,N}(t)\Big]^{(j-1)} * \Big[\int_0^t \overline{F}(u)dA(u)\Big] * \sum_{k=1}^{\infty}\Big[p\int_0^t \overline{B}(u)\overline{U}(u)dF(u)\Big]^{(k-1)}$$
$$*\Big\{(1-p)\int_0^t \overline{B}(u)\overline{U}(u)dF(u) + \int_0^t \overline{F}(u)\overline{B}(u)dU(u)\Big\}, \qquad (A5.4)$$

where the asterisk mark denotes the Stieltjes convolution, $a^{(n)}(t)$ denotes the n-fold Stieltjes convolution of a distribution $a(t)$ with itself and $a^{(0)}(t) \equiv 1$ for $t \geq 0, 0$ for $t < 0$, i.e., $a^{(n)}(t) \equiv a^{(n-1)}(t) * a(t)$, $a(t) * b(t) \equiv \int_0^t b(t-u)da(u)$. For example, $Q_{N,F}(t)$ is the probability distribution that when the N-th μP unit is operating, the system transits to failure state until time t because one of the following three cases: (i) Errors of a μP occur before an initial processing finishes, (ii) a WDP cannot detect errors of μP, and (iii) one processing of μP does not finish until a limit processing time.

Chapter 6

Reliability of a Multi-Microprocessor System with Complicated Switching

This chapter considers a system with N TMR (Triple Modular Redundancy) units in which each unit consists of microprocessor and watchdog processor, and a faulty TMR unit is switched over to a new one. The mean time to system failure and the expected cost are derived, using Markov renewal processes. Optimal numbers N^* of TMR units which maximize the mean time and minimize the expected cost are analytically discussed. Finally, numerical examples are given.

6.1 Introduction

In this chapter, we consider the following system with N TMR (Triple Modular Redundancy) units to improve its reliability: A μP unit consists of microprocessor (μP) and watchdog processor (WDP), and each TMR unit consists of three μP units with majority voting function. When errors of μPs have occurred, a WDP detects them with a certain probability and resets a μP to its initial state. This probability p is called *coverage of a WDP*. Three μP units of a TMR unit make the same one processing, and compare the results with each other at a specified time T. This is automatically switched over to a new TMR unit in standby in the following three cases: (i) More than two results do not agree, (ii) more than two processings are not completed until time T, or (iii) one μP unit becomes faulty.

It has been well-known that even if a system consists of redundant units, its reliability often decreases because the quantities of hardware such as detecting faults and switching circuits increase [Nanya91]. In this chapter, we regard the increase of units as that of complexity, and introduce the measure of system complexity where its reliability decreases as the number of units increases.

We derive the mean time and the expected cost until system failure, using the theory of Markov renewal processes [Osaki92]. Optimal numbers N^* of TMR units which maximize the mean time and minimize the expected cost are analytically discussed. Numerical examples are given and some useful discussions for these results are made.

6.2 Model and Analysis

A μP unit consists of μP and WDP, and the outline of the model is drawn in Figure 6.1.

6.2.1 Analysis of a μP unit

A μP unit repeats one processing which needs a random time according to an exponential distribution $G(t) \equiv 1 - e^{-\mu t}$. We assume that:

(1) Errors of a μP occur according to an exponential distribution $F(t) \equiv 1 - e^{-\lambda t}$.

(2) A WDP can detect errors of a μP with probability p $(0 < p \leq 1)$ and resets a μP to its initial state.

- **(a)** If a WDP cannot detect errors with probability $(1-p)$, a μP becomes faulty.
- **(b)** Reset times are neglected.
- **(c)** A WDP works independently of a μP and does not fail.

Under the above assumptions, we define the following states of a μP unit:

State 0: A μP begins to operate.

State S: A μP completes one processing.

State E: A μP becomes faulty.

The states defined above form a Markov renewal process where both states S and E are an absorbing state.

Let $Q_{0,j}(t)(j = 0, S, E)$ be one-step transition probabilities of a Markov renewal process. Then, we have following equations:

$$Q_{0,0}(t) = p\int_0^t \overline{G}(u)dF(u), \tag{6.1}$$

$$Q_{0,S}(t) = \int_0^t \overline{F}(u)dG(u), \tag{6.2}$$

$$Q_{0,E}(t) = (1-p)\int_0^t \overline{G}(u)dF(u). \tag{6.3}$$

From equations (6.1) $\sim$ (6.3), the transition probabilities $P_{0,j}(t)$ that it is in state j ($j = 0, S, E$) at time t when a μP unit is in state 0 at time 0 are given by

$$P_{0,0}(t) = 1 - Q_{0,0}(t) - Q_{0,S}(t) - Q_{0,E}(t) + Q_{0,0}(t) * P_{0,0}(t), \tag{6.4}$$

$$P_{0,S}(t) = Q_{0,S}(t) + Q_{0,0}(t) * P_{0,S}(t), \tag{6.5}$$

$$P_{0,E}(t) = Q_{0,E}(t) + Q_{0,0}(t) * P_{0,E}(t), \tag{6.6}$$

where the asterisk mark denotes Stieltjes convolution, i.e., $A(t) * B(t) \equiv \int_0^t B(t-u)dA(u)$. Then, arranging above equations, we have the following equations:

$$P_{0,0}(t) = e^{-[\lambda(1-p)+\mu]t}, \tag{6.7}$$

$$P_{0,S}(t) = \frac{\mu}{\lambda(1-p)+\mu}(1 - e^{-[\lambda(1-p)+\mu]t}), \tag{6.8}$$

$$P_{0,E}(t) = \frac{(1-p)\lambda}{\lambda(1-p)+\mu}(1 - e^{-[\lambda(1-p)+\mu]t}), \tag{6.9}$$

where it is evident that $P_{0,0}(t) + P_{0,S}(t) + P_{0,E}(t) = 1$.

6.2.2 Analysis of N TMR units

Each μP unit of a TMR unit repeats the same processing, and has to complete one processing until a specified time T to compare its result. It is assumed that the probability which the result of μP unit is correct is $\alpha(0 < \alpha \leq 1)$. It is judged by the voter of a TMR unit that if more than two results are correct, they are correct, and otherwise, they are not. The system consists of N TMR units where one is operating and the others are in standby.

(3) If more than two results do not agree, one unit becomes faulty, or more than two processings are not completed until time T, then an operating TMR unit is switched over to one of other units in standby.

(4) If more than two μP units are faulty at time T or if a faulty TMR unit cannot be switched over to one of standby units, then the system becomes failure.

The quantities of hardware of detecting faults and switching circuits would increase in proportion to the number N of TMR units. That is, the quantities of hardware of a whole system increases by those of detecting faults and switching circuits, adding to the number of TMR units. In this chapter, we define $V(N)$ as the measure of complexity, which is given by the reliability of a TMR unit and the increased quantities of hardware [IS76].

Let R_u be the reliability of a TMR unit and a $(a \geq 0)$ be the rate of quantities of hardware of detecting faults and switching circuits for those of a TMR unit. Then, we assume that the reliability of complexity for N TMR units is $V(N) \equiv (R_u^a)^N (N = 1, 2, \cdots)$. Evidently, when both N and a increase, $V(N)$ decreases, and hence, the mean time to system failure decreases.

Under the above assumptions, we define the following states of the system:

State i: The i-th TMR unit begins to operate $(i = 1, 2, \cdots, N)$.

State F: System failure occurs.

The transition probabilities $q_{i,j}$ from state i to state j of the system states above are given by the following equations:

$$q_{i,i+1} = \sum_{k=1}^{\infty}[A(T)]^{k-1}[1-A(T)-B(T)]V(N) \quad (i=1,2,\cdots,N-1), \tag{6.10}$$

$$q_{i,F} = \sum_{k=1}^{\infty}[A(T)]^{k-1}\{B(T)+[1-A(T)-B(T)]\overline{V}(N)\} \quad (i=1,2,\cdots,N-1), \tag{6.11}$$

$$q_{N,F} = \sum_{k=1}^{\infty}[A(T)]^{k-1}[1-A(T)], \tag{6.12}$$

where

$$A(T) \equiv [\alpha^3+3\alpha^2(1-\alpha)][P_{0,S}(T)]^3+3\alpha^2[P_{0,S}(T)]^2P_{0,0}(T), \tag{6.13}$$

$$B(T) \equiv 3[1-P_{0,E}(T)][P_{0,E}(T)]^2+[P_{0,E}(T)]^3, \tag{6.14}$$

and $A(T)$ is the probability that a TMR unit completes one processing correctly at time T, $B(T)$ is the probability that more than two units are in faulty state at time T, and $\overline{V}(N) \equiv 1-V(N)$.

6.2.3 Mean time to system failure

We derive the mean time $\ell_F(N)$ from the beginning of system operation to system failure. The expected processing number $M_{i,F}$ of a TMR unit until transition from state i at time 0 to state F without transition to other states is given by

$$\begin{aligned} M_{i,F} &= \sum_{k=1}^{\infty}k[A(T)]^{k-1}\{B(T)+[1-A(T)-B(T)]\overline{V}(N)\} \\ &= \frac{B(T)+[1-A(T)-B(T)]\overline{V}(N)}{[1-A(T)]^2} \quad (i=1,2,\cdots,N-1), \end{aligned} \tag{6.15}$$

$$\begin{aligned} M_{N,F} &= \sum_{k=1}^{\infty}k[A(T)]^{k-1}[1-A(T)] \\ &= \frac{1}{1-A(T)}. \end{aligned} \tag{6.16}$$

Hence, the mean time $\ell_F(N)$ to system failure is

$$\begin{aligned}
\ell_F(N) &= \sum_{j=1}^{N} jT\left(\prod_{i=1}^{j-1} q_{i,i+1}\right)M_{j,F} \\
&= \frac{T}{1-A(T)}\sum_{j=1}^{N}[DV(N)]^{j-1} \quad (N=1,2,\cdots),
\end{aligned} \tag{6.17}$$

where $\prod_{i=1}^{0} \equiv 1$ and $D \equiv [1-A(T)-B(T)]\,/[1-A(T)]$ which is the probability that a TMR unit is switched over to one of standby units at time T.

6.3 Optimal Policy

Suppose that $R_u^a = e^{-\beta}$ and $V(N) = e^{-\beta N}(\beta \geq 0)$, where $\beta = a\ln(1/R_u)$ is a parameter of complexity and represents the failure rate of switching. Then, we discuss an optimal number N^* which maximizes $\ell_F(N)$ in (6.17).

We put formally that

$$\begin{aligned}
\widetilde{\ell_F}(N) &\equiv \frac{1-A(T)}{T}\ell_F(N) \\
&= \sum_{j=1}^{N}(De^{-\beta N})^{j-1},
\end{aligned} \tag{6.18}$$

and seek N^* which maximizes $\widetilde{\ell_F}(N)$. It is evident that for $\beta > 0$,

$$\widetilde{\ell_F}(1) = 1, \tag{6.19}$$

$$\widetilde{\ell_F}(\infty) \equiv \lim_{N\to\infty}\frac{1-D^N e^{-\beta N^2}}{1-De^{-\beta N}} = 1. \tag{6.20}$$

From the inequality $\widetilde{\ell_F}(N) \geq \widetilde{\ell_F}(N+1)$, we have

$$\frac{1}{D^N e^{-\beta N(N+1)}}\sum_{j=1}^{N}\{(De^{-\beta N})^{j-1} - [De^{-\beta(N+1)}]^{j-1}\} \geq 1. \tag{6.21}$$

Denoting the left side of (6.21) by $L_1(N)$, we have

$$L_1(N) - L_1(N-1) = \frac{1}{D^N e^{-\beta N(N+1)}}\sum_{j=1}^{N-1}(De^{-\beta N})^j[1-e^{-\beta j}-e^{-\beta(N-j+1)}+e^{-\beta N}]$$

$$> \frac{1}{D^N e^{-\beta N(N+1)}} \sum_{j=1}^{N-1} (De^{-\beta N})^j (1-e^{-\beta j})[1-e^{-\beta(N-j)}] > 0, \quad (6.22)$$

and

$$\begin{aligned} L_1(1) &= 0, & (6.23) \\ L_1(\infty) &= \lim_{N\to\infty} \frac{1}{D^N e^{-\beta N(N+1)}} \sum_{j=1}^{N} \{(De^{-\beta N})^{j-1} - [De^{-\beta(N+1)}]^{j-1}\} \\ &\geq \lim_{N\to\infty} e^{\beta N(N+1)}[e^{-\beta N} - e^{-\beta(N+1)}] = \infty. & (6.24) \end{aligned}$$

Thus, there exists a finite and unique minimum $N^*(1 \leq N^* < \infty)$ which satisfies (6.21).

Next, we discuss an optimal policy which minimizes the expected cost. Let c_1 be the cost for system failure and c_2 be the cost for a TMR unit. Then, the expected cost $C(N)$ per unit of time of the system with N TMR units is given by

$$C(N) \equiv \frac{c_1 + Nc_2}{\ell_F(N)}. \quad (6.25)$$

We seek an optimal number N^* which minimizes $C(N)$ in (6.25). From the inequality $C(N+1) - C(N) \geq 0$, we have

$$\frac{1}{D^N e^{-\beta N(N+1)}} \left[(\frac{c_1}{c_2} + N) \sum_{j=0}^{N-1} \{(De^{-\beta N})^j - [De^{-\beta(N+1)}]^j\} + \sum_{j=0}^{N-1} (De^{-\beta N})^j \right] - N \geq \frac{c_1}{c_2}. \quad (6.26)$$

Denoting the left side of (6.26) by $L_2(N)$, we have

$$\begin{aligned} &L_2(N) - L_2(N-1) \\ &= \frac{1}{D^N e^{-\beta N(N+1)}} \Big\{ (\frac{c_1}{c_2} + N) \Big[\sum_{j=0}^{N-2} (De^{-\beta N})^j (1-e^{-\beta j})(1-De^{-\beta(2N-j)}) \\ &\quad + (De^{-\beta N})^{N-1}(1-e^{-\beta(N-1)}) \Big] + \sum_{j=0}^{N-1} (De^{-\beta N})^j (1-De^{-2\beta N}) \Big\} > 0, \quad (6.27) \end{aligned}$$

and

$$\begin{aligned} L_2(1) &= \frac{1-De^{-2\beta}}{De^{-2\beta}}, & (6.28) \\ L_2(\infty) &\equiv \lim_{N\to\infty} L(N) = \infty. & (6.29) \end{aligned}$$

Hence, $L_2(N)$ is strictly increasing in N from $L_2(1)$ to ∞. Thus, we have the following optimal policy:

(i) If $L_2(1) < c_1/c_2$, then there exists a finite and unique minimum $N^*(> 1)$ which satisfies (6.26).

(ii) If $L_2(1) \geq c_1/c_2$, then $N^* = 1$.

Finally, we consider the special case where $\beta = 0$ and $V(N) = 1$, i.e., we do not consider the complexity of system. In this case, the mean time to failure is

$$\ell_F(N) = \frac{T}{B(T)}(1 - D^N), \tag{6.30}$$

which is strictly increasing in N. Hence an optimal N^* which maximizes $\ell_F(N)$ tends to infinity.

Further, the expected cost is

$$C(N) = \frac{B(T)}{T}\frac{c_1 + Nc_2}{1 - D^N}. \tag{6.31}$$

From $C(N+1) \geq C(N)$, we have

$$\frac{1}{D^N}\sum_{j=0}^{N-1} D^j - N \geq \frac{c_1}{c_2}. \tag{6.32}$$

The left side of (6.32) is strictly increasing from $1/D - 1$ to ∞. Thus, there exists a finite and unique minimum $N^*(1 \leq N^* < \infty)$ which satisfies (6.32).

6.4 Numerical Examples

We compute numerically the mean time $\ell_F(N)$ and the optimal number N^* which minimizes $C(N)$. Suppose that the mean processing time $1/\mu$ of μP is a unit of time of the system and the mean time to error occurrences is $(1/\lambda)/(1/\mu) = 3600 \times 24$. Further, the coverage of a WDP is $p = 0.8 \sim 0.99$, the probability that the processing result is correct is $\alpha = 0.999$ and the cost rate of system failure for a TMR unit is $c_1/c_2 = 1 \sim 5$.

Table 6.1 gives the optimal number N^* which minimizes $C(N)$ when $p = 0.9, \mu T = 10$. For example, when $\beta = 0.05, c_1/c_2 = 2$, the optimal number of TMR units is $N^* = 3$. This indicates that N^* decreases with β, i.e., the number of TMR units has to be small as the system becomes more complex. However, even if the system becomes more redundant, it is economical for small β. Further, N^* increases with c_1/c_2. That is, if the cost of system failure increases, N^* has to be large to prevent system failure.

Table 6.2 gives the mean time to failure $\ell_F(N)$ for N and β when $p = 0.9$ and $\mu T = 10$, where an asterisk mark denotes the maximum value for each β. For example, when $\beta = 0.1$, the mean time to system failure reaches a maximum at $N = 4$, and then, $\ell_F(4) = 3.587 \times 10^6$. This indicates that $\ell_F(N)$ decreases with β for the same N, and optimal N^* which maximizes $\ell_F(N)$ also decreases with β. This has the same tendency as that of Table 6.1. When $c_1 = 1$ and $c_2 = 0$ in (6.25), $C(N) = 1/\ell_F(N)$, and hence, the optimal policy which minimizes $C(N)$ is equal to the same problem which maximizes $\ell_F(N)$. It is of interest that optimal N^* of Table 6.2 corresponds to that of Table 6.1 for $c_1/c_2 = \infty$, and gives an upper limit number of TMR units.

Figures 6.2 and 6.3 draw $\ell_F(3)$ for μT when $p = 0.8, 0.9, 0.99, \beta = 0.05$, and $p = 0.9, \beta = 10^{-1}, 10^{-2}, 10^{-3}$, respectively. Figure 6.2 indicates that $\ell_F(3)$ increases with $p, \mu T$, however, Figure 6.3 indicates that it decreases with β and nearly converges to the value of $\beta = 10^{-3}$. It is easily seen that the coverage p gives a greater influence on the mean time than β. Hence, to develop the reliability of the system, we should more improve the coverage of a WDP. On the other hand, we can also estimate the coverage p and the parameter β of complexity from Figures 6.2 and 6.3, respectively, when the processing limit time μT and the mean time $\ell_F(N)$ are given.

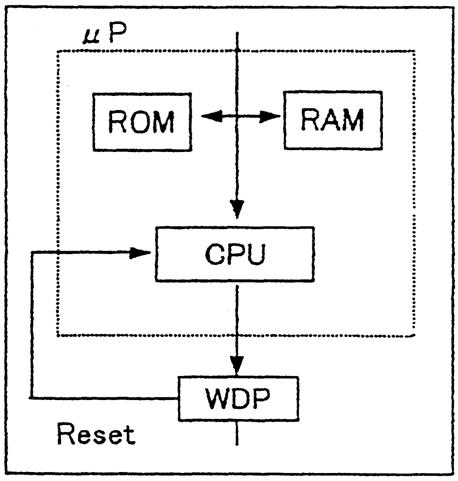
Further, noting that $\beta = a \ln(1/R_u)$, we can see in Table 6.1 that when R_u increases, β decreases, and hence, N^* becomes large. Similarly, when a increases, β also increases and N^* becomes small.

6.5 Conclusions

We have considered the reliability of a system with N TMR units, and have derived the mean time to system failure and the expected cost. We have introduced the concept of complexity, from the viewpoint of complicated switching of the system, and have discussed optimal numbers of TMR units. It has been shown from the numerical examples that the optimal number decreases with the parameter β of complexity, and increases with the cost rate c_1/c_2 of system failure.

Further, it has been shown that the optimal number decreases with the rate of quantities of hardware of detecting faults and switching circuits for those of a TMR unit, and increases with the reliability of a TMR unit. Thus, we could design more redundant systems with high reliability as the reliability of each unit develops and the complexity becomes small.

A μ P unit



The system with N TMR units

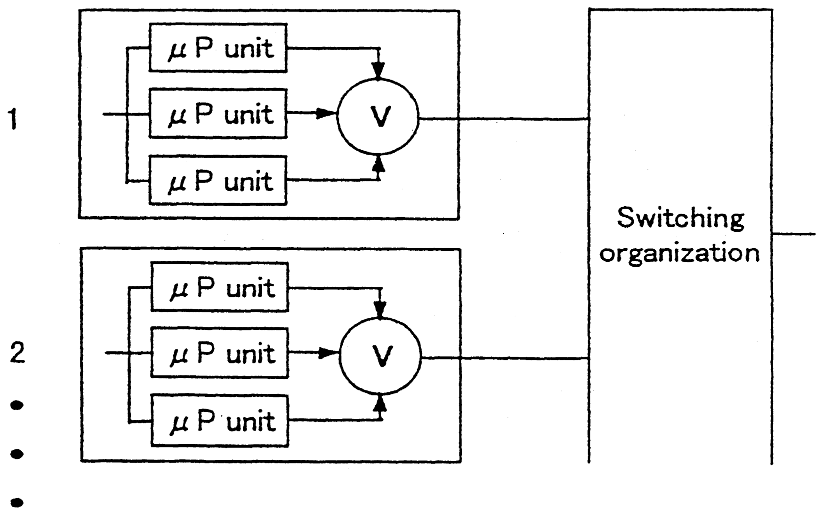


Figure 6.1: Outline of the model.

Table 6.1: Optimal number N^* to minimize $C(N)$.

β	c_1/c_2				
	1	2	3	4	5
0.01	4	6	6	7	7
0.02	4	4	5	5	6
0.05	3	3	4	4	4
0.1	2	3	3	3	3

Table 6.2: Mean time to failure $\ell_F(N)$.

($\times 10^6$)

N	β				
	0	0.01	0.02	0.05	0.1
1	1.482	1.482	1.482	1.482	1.482
2	2.964	2.934	2.906	2.823	2.695
3	4.446	4.316	4.192	3.855	3.393
4	5.928	5.588	5.278	4.502	*3.587
5	7.410	6.721	6.127	*4.780	3.457
6	8.891	7.693	6.726	4.773	3.195
7	10.373	8.491	7.086	4.585	2.922
8	11.855	9.111	*7.236	4.312	2.687
9	13.337	9.558	7.216	4.018	2.497
10	14.819	9.844	7.069	3.741	2.344
11	16.301	9.984	6.837	3.495	2.221
12	17.783	*10.000	6.555	3.282	2.121
13	19.265	9.913	6.252	3.100	2.037
14	20.747	9.745	5.948	2.944	1.967
15	22.229	9.518	5.654	2.809	1.908

* : maximum value of $\ell_F(N)$

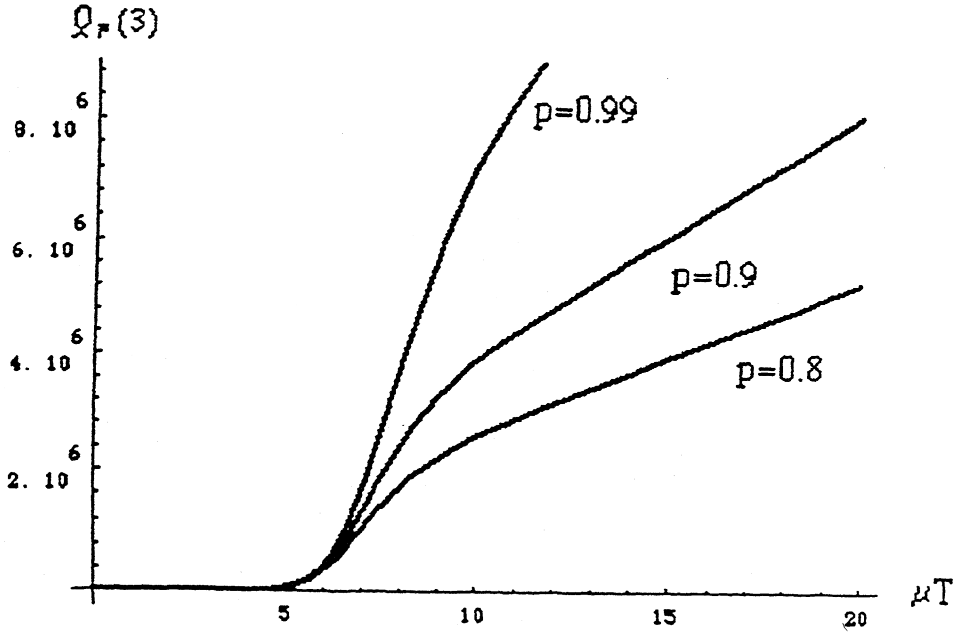


Figure 6.2: Mean time $\Omega_F(3)$ to failure when β=0.05.

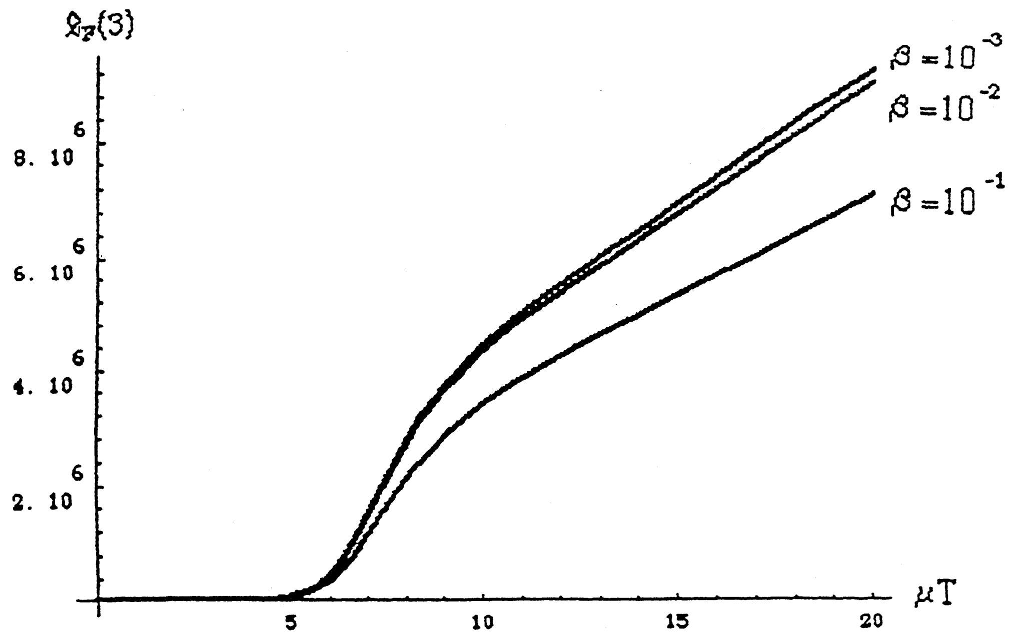


Figure 6.3: Mean time $\ell_F(3)$ to failure when p=0.9.

Chapter 7

Optimal Reset Number of a Microprocessor System with Network Processing

This chapter considers the maintenance problem for improving the reliability of a microprocessor (μP) system with network processing. After the system has made a stand-alone processing, it executes successively a communication procedure of a network processing. When either μP failures or application software errors in the system have occurred, a μP is reset to the beginning of its initial state and restarts again. The reliability quantities such as the mean time to success of a network processing and the expected reset number are derived, using the theory of Markov renewal processes. An optimal reset number, which minimizes the expected cost until a network processing is successful, is analytically discussed. Numerical examples are finally given.

7.1 Introduction

As a computer network technology has remarkably developed, microcomputers (μPs) which form a data terminal equipment (DTE) in a communication network have been used in many practical fields. Recently, a new communication network combining the information processing and communication has played an important role as the infrastructure in the information society has developed. Therefore, the demand for improvement of reliabilities and functions for devices of a communication network have greatly increased[Ono96, Akiyama97].

In fact, a μP which is one of vital devices of a communication network often fails through some faults due to noise, changes in the environment and programming bugs. Hence, it is necessary to make the preventive maintenance for occurrences of such errors. Generally, when we consider the reliability of the system on an operational stage, we should regard the cause of error occurrences of a μP as faults of software, such as mistakes of operational control and memory access, rather than faults of hardware. That is, when errors of a μP have occurred, it would be effective to recover the system by the operation of reset [Nanya91].

This chapter considers the maintenance problem for improving the reliability of a μP system with network processing: After the system has made a stand-alone processing, it executes successively communication procedures of a network processing. When either μP failures or application software errors in the system have occurred, a μP is reset to the beginning of its initial state and restarts again. Most reliability evaluation models of a μP system until now have assumed that both errors of a μP and failures of the data transmission occur unlimitedly [YMN91, YNM92, SNK92, NYS93, YNS95]. This chapter assumes that if the reset due to errors has occurred N times intermittently, then a μP interrupts its processing and restarts again from the beginning of its initial state after a constant time. That is, if the reset has occurred frequently, the system has latent faults, and makes the preventive maintenance to check the operational environment and to eliminate errors.

We derive the reliability quantities such as the mean time and the expected reset number until a network processing is successful. Further, we regard the losses which are the times for the reset and the interruption of processing and for the maintenance to restart the system as expected costs, and discuss optimal policies which minimize them. Numerical examples are finally given.

7.2 Model and Analysis

We pay attention to only a certain DTE which consists of a workstation or a personal computer and connects with some networks, and consider the problem for improving its reliability.

Suppose that errors of a μP system occur according to an exponential distribution $F(t) \equiv 1 - e^{-\lambda t}$. If errors of a μP have occurred, a μP is reset to the beginning of its initial state and restarts again. It is assumed that any reset times are neglected.

(1) After a μP begins to operate, it executes an initial processing immediately and a stand-alone processing.

(2) The times for an initial processing and a stand-alone processing have a general distribution $V(t)$ with finite mean $1/v$ and an exponential distribution $A(t) \equiv 1 - e^{-\alpha t}$, respectively.

(3) After a μP completes a stand-alone processing, it begins to execute a network connection processing:

- **(a)** A connection processing needs the time according to a general distribution $B(t)$ with finite mean $1/\beta$ and fails with probability γ $(0 \leq \gamma < 1)$.
- **(b)** If a connection processing has failed, a μP executes the same processing again after a constant time w where $W(t) \equiv 0$ for $t < w$ and 1 for $t \geq w$.

(4) After a connection processing has been successful, a μP executes a network processing.

- **(c)** A network processing needs the time according to a general distribution $U(t)$ with finite mean $1/u$, and is successful with probability 1 if it has not failed.

(5) If the N-th reset has occurred since a μP begins to operate, once it interrupts the processing, and restarts again from the beginning after a constant time μ, where $G(t) \equiv 0$ for $t < \mu$ and 1 for $t \geq \mu$.

Under the above assumptions, we define the following states of the system:

State 0: An initial processing begins.

State 1: A stand-alone processing begins.

State 2: A stand-alone processing is completed and a network connection processing begins.

State 3: A network connection processing succeeds and a network processing begins.

State F: A processing is interrupted.

State S: A network processing succeeds.

The system states defined above form a Markov renewal process [Osaki92] where state S is an absorbing state. Transition diagram between system states is shown in Figure 7.1.

Let $Q_{i,j}(t)$ $(i = 0, 1, 2, 3; j = 0, 1, 2, 3, S)$ be one-step transition probabilities of a Markov renewal process. Then, mass functions $Q_{i,j}(t)$ from state i at time 0 to state j at time t are:

$$Q_{0,0}(t) = \int_0^t \overline{V}(u)dF(u), \tag{7.1}$$

$$Q_{0,1}(t) = \int_0^t \overline{F}(u)dV(u), \tag{7.2}$$

$$Q_{1,0}(t) = \int_0^t \overline{A}(u)dF(u), \tag{7.3}$$

$$Q_{1,2}(t) = \int_0^t \overline{F}(u)dA(u), \tag{7.4}$$

$$Q_{2,0}(t) = \sum_{j=1}^{\infty} X^{(j-1)}(t) * \int_0^t [\overline{B}(u) + \gamma B(u) * \overline{W}(u)]dF(u), \tag{7.5}$$

$$Q_{2,3}(t) = \sum_{j=1}^{\infty} X^{(j-1)}(t) * [(1-\gamma)\int_0^t \overline{F}(u)dB(u)], \tag{7.6}$$

$$Q_{3,0}(t) = \int_0^t \overline{U}(u)dF(u), \tag{7.7}$$

$$Q_{3,S}(t) = \int_0^t \overline{F}(u)dU(u), \tag{7.8}$$

where

$$X(t) \equiv \gamma \int_0^t \overline{F}(u)dB(u) * \int_0^t \overline{F}(u)dW(u), \tag{7.9}$$

the asterisk mark denotes the Stieltjes convolution and $a^{(n)}(t)$ denotes the $n-$fold Stieltjes convolution of a distribution $a(t)$ with itself, i.e., $a^{(n)}(t) \equiv a^{(n-1)}(t) * a(t)$, $a(t) * b(t) \equiv \int_0^t b(t-u)da(u)$.

We derive the mean time ℓ_S from the beginning of system operation until a network processing is successful. Let $H_{0,S}(t)$ be the first-passage time distribution from state 0 to state S. Then, we have

$$H_{0,S}(t) = \sum_{j=1}^{N} D^{(j-1)}(t) * Z(t), \tag{7.10}$$

where

$$\begin{aligned} D(t) &\equiv Q_{0,0}(t) + Q_{0,1}(t) * Q_{1,0}(t) + Q_{0,1}(t) * Q_{1,2}(t) * Q_{2,0}(t) \\ &\quad + Q_{0,1}(t) * Q_{1,2}(t) * Q_{2,3}(t) * Q_{3,0}(t), \end{aligned} \tag{7.11}$$

$$Z(t) \equiv Q_{0,1}(t) * Q_{1,2}(t) * Q_{2,3}(t) * Q_{3,S}(t). \tag{7.12}$$

It is noted that $D(t)$ is the distribution function which a μP is reset by the occurrence of errors and $Z(t)$ is the distribution function which the system moves from state 0 to state F directly without being reset. Further, the first-passage time distribution $H_{0,F}(t)$ from state 0 to state F by the N-th reset of a μP is given by

$$H_{0,F}(t) = D^{(N)}(t). \tag{7.13}$$

Therefore, the first-passage time distribution $L_S(t)$ until a network processing is successful is given by the following renewal equation:

$$L_S(t) = H_{0,S}(t) + H_{0,F}(t) * G(t) * L_S(t). \tag{7.14}$$

Let $\phi(s)$ be the Laplace-Stieltjes (LS) transform of any function $\Phi(t)$, i.e., $\phi(s) \equiv \int_0^\infty e^{-st} d\Phi(t)$. Taking the LS transforms on both sides of (7.14) and arranging them, we have

$$l_S(s) = \frac{h_{0,S}(s)}{1 - h_{0,F}(s)g(s)}. \tag{7.15}$$

Hence, the mean time ℓ_S is given by

$$\begin{aligned} \ell_S &\equiv \int_0^\infty t dL_S(t) = \lim_{s \to 0}[-\frac{dl_S(s)}{ds}] \\ &= -\frac{z'(0) + d'(0)}{1 - d(0)} + \frac{\mu d(0)^N}{1 - d(0)^N}, \end{aligned} \tag{7.16}$$

where $\phi'(s)$ is the differential function of $\phi(s)$, i.e., $\phi'(s) \equiv d\phi(s)/ds$. From equation (7.16), it is noted that ℓ_S is strictly decreasing in N and is minimized when $N = \infty$.

Next, we derive the expected reset number M_R from the start of system operation or the restart by the reset until a network processing is successful. Let $M_R(t)$ be the expected reset number until a network processing is successful in an interval $(0, t]$. Then, we have

$$M_R(t) = \sum_{j=1}^{N-1} jD^{(j)}(t) * Z(t). \tag{7.17}$$

Thus, the expected reset number is given by

$$\begin{aligned} M_R &\equiv \lim_{t \to \infty} M_R(t) = \lim_{s \to 0} \sum_{j=1}^{N-1} j[d(s)]^j z(s) \\ &= \frac{d(0)}{1 - d(0)}[1 - Nd(0)^{N-1} + (N-1)d(0)^N], \end{aligned} \tag{7.18}$$

where it is noted that $z(0) = 1 - d(0)$.

Further, let $M_F(t)$ be the distribution of the expected interruption number of processings from the start of system operation until a network processing is successful. Then, we have the following renewal equation:

$$M_F(t) = H_{0,F}(t) * [1 + G(t) * M_F(t)]. \tag{7.19}$$

Similarly, the expected interruption number M_F until a network processing is successful is given by

$$M_F = \frac{d(0)^N}{1-d(0)^N}. \tag{7.20}$$

7.3 Optimal Policies

We obtain two objective functions which are the total expected cost $C_1(N)$ and the expected cost $C_2(N)$ per unit of time until a network processing is successful, and discuss optimal policies which minimize them, respectively.

7.3.1 Policy 1

Let c_1 be the cost for the reset and c_2 be the cost for an interruption of processing. Then, we define the total expected cost $C_1(N)$ until a network processing is successful as the following equation:

$$\begin{aligned} C_1(N) &\equiv c_1 M_R + c_2 M_F \\ &= c_1[\frac{D(1-D^N)}{1-D} - ND^N] + \frac{c_2 D^N}{1-D^N} \quad (N=1,2,\cdots), \end{aligned} \tag{7.21}$$

where $D \equiv d(0)$ is the probability that a μP is reset.

We seek an optimal number N_1^* which minimizes $C_1(N)$. From the inequality $C_1(N+1) - C_1(N) \geq 0$, we have

$$N(1-D^N)(1-D^{N+1}) \geq \frac{c_2}{c_1}. \tag{7.22}$$

Denoting the left-hand side of (7.22) by $L_1(N)$, we have

$$L_1(1) = (1-D)(1-D^2), \tag{7.23}$$

$$L_1(\infty) \equiv \lim_{N\to\infty} L_1(N) = \infty. \tag{7.24}$$

Hence, $L_1(N)$ is strictly increasing in N from $L_1(1)$ to ∞. Thus, we have the following optimal policy:

(i) If $L_1(1) < c_2/c_1$, then there exists a finite and unique minimum $N_1^*(> 1)$ which satisfies (7.22).

(ii) If $L_1(1) \geq c_2/c_1$, then $N_1^* = 1$ and the total expected cost is $C_1(1) = (c_2 D)/(1-D)$.

In this model, c_1 is the cost for the increase of system resources such as spaces of memory and times by the reset, and c_2 is the cost for the increase of system resources by the preventive maintenance to eliminate the cause of errors. It could be generally estimated that c_2 is greater than c_1, i.e., $c_2 \geq c_1$. Thus, we have $L_1(1) < c_2/c_1$, and hence, $N_1^* > 1$. Further, it is easily shown that N_1^* increases with c_2/c_1.

7.3.2 Policy 2

In the policy 1, we have adopted the total expected cost as an objective function. However, it would be more practical to introduce the measure of the time until a network processing is successful. Next, we consider an optimal policy which minimizes the expected cost per unit of time until a network processing is successful. That is, from equations (7.16) and (7.21), we define the expected cost $C_2(N)$ per unit of time as the following equation:

$$\begin{aligned} C_2(N) &\equiv \frac{C_1(N)}{\ell_S} \\ &= \frac{c_1[\frac{D(1-D^N)}{1-D} - ND^N] + c_2\frac{D^N}{1-D^N}}{A + \frac{\mu D^N}{1-D^N}} \\ &= \frac{c_1\sum_{j=1}^{N-1} jD^j(1-D) - \frac{A}{\mu}c_2}{A + \frac{\mu D^N}{1-D^N}} + \frac{c_2}{\mu} \quad (N = 1, 2, \cdots), \end{aligned} \tag{7.25}$$

where

$$A \equiv -\frac{z'(0) + d'(0)}{1-D} > 0. \tag{7.26}$$

We seek an optimal number N_2^* which minimizes $C_2(N)$. From the inequality $C_2(N+1) - C_2(N) \geq 0$, we have

$$N(1-D^N)(1-D^{N+1})+\frac{\mu}{A}[ND^N(1-D^{N+1})+(1-D)\sum_{j=1}^{N-1} jD^j] \geq \frac{c_2}{c_1}. \tag{7.27}$$

Denoting the left-hand side of (7.27) by $L_2(N)$,

$$\begin{aligned} L_2(1) &= (1-D^2)(1-D+\frac{\mu}{A}D), & (7.28)\\ L_2(\infty) &\equiv \lim_{N\to\infty} L_2(N)=\infty. & (7.29)\end{aligned}$$

Putting the second term on the bracket of the left-hand side of (7.27) by

$$L_3(N) \equiv ND^N(1-D^{N+1})+(1-D)\sum_{j=1}^{N-1} jD^j, \tag{7.30}$$

we have

$$\begin{aligned} L_3(1) &= (1-D^2)D, & (7.31)\\ L_3(N+1)-L_3(N) &= D^{N+1}[1-D^{N+2}+ND^N(1-D^2)]>0. & (7.32)\end{aligned}$$

Hence, $L_3(N)$ is strictly increasing in N. Further, since $N(1-D^N)(1-D^{N+1})$ in (7.27) is also strictly increasing in N, $L_2(N)$ is also strictly increasing in N from $L_2(1)$ to ∞. Thus, we have the following optimal policy:

(i) If $L_2(1) < c_2/c_1$, then there exists a finite and unique minimum $N_2^*(>1)$ which satisfies (7.27).

(ii) If $L_2(1) \geq c_2/c_1$, then $N_2^*=1$, and the resulting cost is

$$C_2(1)=\frac{c_2 D}{A(1-D)+\mu D}. \tag{7.33}$$

Further, we compare the optimal policy 2 with the optimal policy 1. Since from equations (7.22) and (7.27),

$$L_2(N) - L_1(N) = \frac{\mu}{A}[ND^N(1 - D^{N+1}) + (1 - D)\sum_{j=1}^{N-1} jD^j] > 0 \qquad (N = 1, 2, \cdots), \tag{7.34}$$

and hence, $N_1^* \geq N_2^*$.

This means that when the number N of reset is small, the mean time until a network processing is successful is large, since ℓ_S strictly decreases in N. Thus, it would be better to adopt Policy 2 where N is small when we consider only the cost of the system on the whole. On the other hand, if we want a processing time to be small, we should adopt Policy 1.

7.4 Numerical Examples

We compute numerically the optimal number N_2^* which minimizes $C_2(N)$ for Policy 2. Suppose that the mean initial processing time $1/v$ of μP is a unit of time and the mean time to error occurrences is $(1/\lambda)/(1/v) = 30 \sim 60$. Further, the mean stand-alone processing time is $(1/\alpha)/(1/v) = 5 \sim 20$, the mean network connection processing time is $(1/\beta)/(1/v) = 1$, the mean waiting time when a network connection processing fails is $w/(1/v) = 1 \sim 4$, the mean network processing time is $(1/u)/(1/v) = 10$, the mean maintenance time after an interruption of processing is $(1/\mu)/(1/v) = 10$, the probability that a network connection processing fails is $\gamma = 0.1, 0.2, 0.4, 0.6$, and the cost c_1 for the reset is a unit of cost and the cost rate of an interruption of processing is $c_2/c_1 = 1 \sim 3$.

Table 7.1 gives the optimal reset number N_2^* which minimizes the expected cost $C_2(N)$. For example, when $(1/\lambda)/(1/v) = 60$, $wv = 2$, $\gamma = 0.2$, $(1/\alpha)/(1/v) = 10$ and $c_2/c_1 = 2$, the optimal number is $N_2^* = 3$. This indicates that the optimal number N_2^* decreases with $(1/\lambda)/(1/v)$, however, increases with wv, γ, $(1/\alpha)/(1/v)$ and c_2/c_1. This can be interpreted that when the cost for an interruption of processing is large, N_2^* increases with c_2/c_1, and so, the processing should not be excessively interrupted.

That is, we should keep on executing the processing as long as possible by the reset. Table 7.1 also shows that N_2^* depends on each parameter when $(1/\lambda)/(1/v)$ is small, i.e., when errors of a μP occur frequently, however, N_2^* depends little on wv, γ and $(1/\alpha)/(1/v)$ when $(1/\lambda)/(1/v) \geq 60$, and in this case, N_2^* is almost determined by c_2/c_1.

7.5 Conclusions

We have investigated the problem for improving the reliability of a μP system with network processing, and have derived the mean time and the expected reset numbers until a network processing is successful. Further, we have discussed optimal reset numbers which minimize the total expected cost and the expected cost per unit of time.

It has been shown from the mathematical analysis that the optimal reset number which minimizes the total cost is larger than that which minimizes the expected cost per unit of time. It has been also shown from the numerical example that the optimal reset number which minimizes the expected cost decreases with the mean time to error occurrences of a μP, however, increases with the mean stand-alone processing time, the probability that a network processing fails and the cost for an interruption of processing. Further, it has been shown that when the mean time to error occurrences is large, the optimal reset number depends little on each parameter and is almost determined by the cost for an interruption of processing.

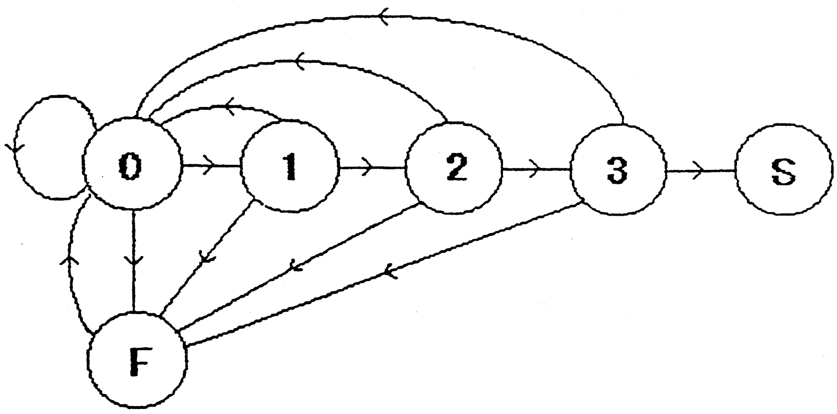


Figure 7.1: Transition diagram between system states.

Table 7.1: Optimal reset number N_2^* to minimize $C_2(N)$.

$(1/\lambda)/(1/v)$	w v	r	$(1/\alpha)/(1/v)=5$					$(1/\alpha)/(1/v)=10$					$(1/\alpha)/(1/v)=20$				
			C_2/C_1					C_2/C_1					C_2/C_1				
			1	2	2	3	3	1	2	2	3	3	1	2	2	3	3
30	1	0.1	2	2	3	3	4	2	3	3	3	4	2	3	3	4	4
		0.2	2	2	3	3	4	2	3	3	3	4	2	3	3	4	4
		0.4	2	2	3	3	4	2	3	3	4	4	2	3	3	4	4
		0.6	2	3	3	3	4	2	3	3	4	4	3	3	4	4	4
	2	0.1	2	2	3	3	4	2	3	3	3	4	2	3	3	4	4
		0.2	2	2	3	3	4	2	3	3	3	4	2	3	3	4	4
		0.4	2	3	3	3	4	2	3	3	4	4	3	3	3	4	4
		0.6	2	3	3	3	4	2	3	3	4	4	3	3	4	4	4
	4	0.1	2	2	3	3	4	2	3	3	3	4	2	3	3	4	4
		0.2	2	2	3	3	4	2	3	3	4	4	2	3	3	4	4
		0.4	2	3	3	3	4	2	3	3	4	4	3	3	4	4	4
		0.6	2	3	3	4	4	2	3	3	4	4	3	3	4	4	5
60	1	0.1	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.2	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.4	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.6	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
	2	0.1	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.2	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.4	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.6	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
	4	0.1	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.2	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.4	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.6	2	2	3	3	4	2	2	3	3	4	2	3	3	3	4

Chapter 8

Reliability of a Job Execution Process Using Signatures

This chapter considers the reliability problem of a microprocessor system whose errors can be detected by using signatures: A system consists of DMR (Double Modular Redundancy) i.e., the same job is executed on two processors. A job is divided into N tasks each of which takes signatures. Signatures are compared at the end of each task. If signatures do not agree, its task executes again. The mean time and the total processing number of tasks until a job completes successfully are derived, using the theory of Markov renewal processes. Moreover, an optimal policy which minimizes the mean time is discussed. Numerical examples show that it is effective to take signatures when the size of a job is large.

8.1 Introduction

As the techniques of error detection of microprocessors (μPs), three checkpoints which compare and store the states, or use signatures have been well-known [Touma90, ZB97, Vaidya98]. A parity check to detect errors is also one kind of signatures. Recently, watchdog processors, which detect errors by comparing signatures and computing results, have been widely used [Nanya91].

This chapter considers the reliability problem of a μP system with signatures: A job is executed on a μP system and is divided into tasks with signatures. If a job is not divided, it has to be executed again from the beginning when some errors have occurred. Consequently, this may incur a job execution time longer. Further, to detect errors of a μP system, it consists of DMR (Double Modular Redundancy), i.e., two processors execute the same job with signatures, which are compared at the end of a task execution. If signatures do not agree, two processors execute again from the beginning of a task execution. If they agree with each other, two processors continue to the next task execution.

We are interested in the number of tasks to reduce a job execution time, by dividing a job into tasks. For this purpose, we obtain the mean execution time to complete a job successfully, using the theory of Markov renewal processes [Osaki92], and discuss an optimal number of tasks which minimizes it. Finally, numerical examples are given, and show that the division with signatures is effective when the size of a job is large.

8.2 Model and Analysis

(1) The system consists of DMR and two processors execute the same job.

(2) A job is divided into N tasks, which take signatures and are executed sequentially. The processing times of each task have a general distribution $A_i(t)(i = 1, 2, \cdots, N)$. Signatures are compared with each other when each task terminates. The comparison time has a general distribution $B(t)$ with finite mean b.

- **(a)** If the signatures are different, the processing result is not correct. In this case, the task executes again after the time which has a general distribution $G(t)$ with finite mean μ.
- **(b)** If the signatures are identical, the next task executes. All processing results of a job are compared after the processing of all tasks have completed. The comparison time has a general distribution $V(t)$ with finite mean v. Its comparison agrees with probability $p(0 < p \leq 1)$ and the processing result

of a job is correct. On the other hand, its comparison does not agree with probability $1-p$ and the processing result is not correct. In this case, a job executes again from the beginning after the time which has a general distribution $W(t)$ with finite mean w.

(3) Errors of a processor in the execution of each task occur independently according to an exponential distribution $(1-e^{-\lambda t})$.

(c) Some errors are detected by the signatures when the processing of each task terminates. Undetected errors are detected finally by comparing all results of a job.

(d) If errors have occurred, the signatures are different.

(4) When all processings of N tasks have completed, a job completes successfully.

Under the above assumptions, we define the following states of the system:

State 0: Processing of a job starts.

State i: Processing of task i completes $(i = 1, 2, \cdots, N)$.

State S: Processing of a job completes successfully.

The states defined above form a Markov renewal process where state S is an absorbing state. Transition diagram between system states is shown in Figure 8.1.

Let $Q_{i,j}(t)$ $(i = 0, 1, \cdots, N; j = 0, 1, \cdots, N, S)$ be one-step transition probabilities of a Markov renewal process. Then, we have the following equations:

$$Q_{i,i}(t) = [\int_0^t (1-e^{-2\lambda u}) dA_i(u)] * B(t) * G(t) \qquad (i = 0, 1, \cdots, N-1), \tag{8.1}$$

$$Q_{i,i+1}(t) = [\int_0^t e^{-2\lambda u} dA_i(u)] * B(u) \qquad (i = 0, 1, \cdots, N-1), \tag{8.2}$$

$$Q_{N,0}(t) = (1-p)V(t) * W(t), \tag{8.3}$$

$$Q_{N,S}(t) = pV(t), \tag{8.4}$$

where the asterisk mark denotes Stieltjes convolution, i.e., $a(t)*b(t) \equiv \int_0^t b(t-u)da(u)$.

First, we derive the mean time $\ell_{0S}(N)$ until a job completes successfully. Let $H_{0S}(t)$ be the first-passage time distribution from state 0 to state S. Then, we have

$$\begin{aligned} H_{0S}(t) &= [\sum_{j=1}^{\infty} Q_{0,0}^{(j-1)}(t) * Q_{0,1}(t)] * [\sum_{j=1}^{\infty} Q_{1,1}^{(j-1)}(t) * Q_{1,2}(t)] * \\ &\quad \cdots * [\sum_{j=1}^{\infty} Q_{N-1,N-1}^{(j-1)}(t) * Q_{N-1,N}(t)] * [Q_{N,S}(t) + Q_{N,0}(t) * H_{0S}(t)], \end{aligned} \tag{8.5}$$

where $a^{(i)}(t)$ denotes the i-fold Stieltjes convolution of a distribution $a(t)$ with itself, i.e., $a^{(i)}(t) \equiv a^{(i-1)}(t) * a(t)$.

Let $\phi(s)$ be the Laplace-Stieltjes(LS) transform of any function $\Phi(t)$ and $\phi'(s)$ be the differential function of $\phi(s)$, i.e., $\phi(s) \equiv \int_0^{\infty} e^{-st} d\Phi(t)$ and $\phi'(s) \equiv d\phi(s)/ds$. Then, the mean time $\ell_{0S}(N)$ is given by

$$\ell_{0S}(N) \equiv \lim_{s \to 0}[-h'_{0S}(s)] = \frac{1}{p}[\sum_{i=0}^{N-1} \ell_i(N) + v + (1-p)w], \tag{8.6}$$

where

$$\ell_i(N) \equiv -\frac{q'_{i,i}(0) + q'_{i,i+1}(0)}{1 - q_{i,i}(0)}, \tag{8.7}$$

which represents the mean processing time of task i.

Next, we derive the total expected processing number $S(N)$ of tasks until a job completes successfully. The expected processing number $S_i(N)$ of task i is given by

$$S_i(N) \equiv \sum_{j=1}^{\infty} j[q_{i,i}(0)]^{j-1} q_{i,i+1}(0) = \frac{1}{1 - q_{i,i}(0)}. \tag{8.8}$$

Thus, the total processing number of tasks is

$$S(N) = \sum_{j=1}^{\infty} j[1-p]^{j-1} p \sum_{i=0}^{N-1} S_i(N) = \frac{1}{p} \sum_{i=0}^{N-1} \frac{1}{1 - q_{i,i}(0)}. \tag{8.9}$$

8.3 Optimal Policy

We discuss an optimal policy which minimizes the mean time $\ell_{0S}(N)$ until a job completes successfully. We seek an optimal division number N^* which minimizes $\ell_{0S}(N)$

in (8.6). From the inequality $\ell_{0S}(N+1) - \ell_{0S}(N) \geq 0$, we have

$$\sum_{i=0}^{N} \ell_i(N+1) - \sum_{i=0}^{N-1} \ell_i(N) \geq 0. \tag{8.10}$$

Denoting the left side of (8.10) by $L(N)$, we have

$$\begin{aligned} L(1) &= \ell_0(2) + \ell_1(2) - \ell_0(1), && (8.11)\\ L(N) - L(N-1) &= \sum_{i=0}^{N} \ell_i(N+1) + \sum_{i=0}^{N-2} \ell_i(N-1) - 2\sum_{i=0}^{N-1} \ell_i(N). && (8.12) \end{aligned}$$

Hence, if $\sum_{i=0}^{N} \ell_i(N+1) + \sum_{i=0}^{N-2} \ell_i(N-1) > 2\sum_{i=0}^{N-1} \ell_i(N)$, then $L(N)$ is strictly increasing in N from $L(1)$.

Thus, we have the following optimal policy:

(i) If $\ell_0(2) + \ell_1(2) < \ell_0(1)$, then there exists a finite and unique minimum $N^*(> 1)$ which satisfies (8.10).

(ii) If $\ell_0(2) + \ell_1(2) \geq \ell_0(1)$, then $N^* = 1$. In this case, we should not divide a job.

The processing times of each task would be random and proportional to the size of a task. Thus, we assume that the processing times of each task have an identical exponential distribution, i.e., $A(t) = 1 - e^{-(N/a)t}$. Then, the mean time $\ell_{0S}(N)$ in (8.6) is

$$\ell_{0S}(N) = \frac{1}{p}[(N + 2\lambda a)(\frac{a}{N} + b) + 2\lambda\mu a + v + (1-p)w], \tag{8.13}$$

and the inequality (8.10) is simply rewritten as

$$N(N+1) \geq \frac{2\lambda a^2}{b}. \tag{8.14}$$

Thus, if $1/\lambda < a^2/b$, then there exists a finite and unique minimum $N^*(> 1)$ which satisfies (8.14). Further, the total processing number of tasks is

$$S(N^*) = \frac{1}{p}(N^* + 2\lambda a). \tag{8.15}$$

8.4 Numerical Examples

We compute numerically the mean time $\ell_{0S}(N)$ and the optimal number N^* which minimizes $\ell_{0S}(N)$. Suppose that the mean comparison time b of signatures is a unit of time of the system, the processing times of each task have an identical exponential distribution $(1 - e^{-(N/a)t})$, the mean processing time when a job is not divided is $a/b = 100 \sim 400$, where its parameter a represents the size of a job. Further, suppose that the mean time to error occurrences is $(1/\lambda)/b = 3600 \sim 18000$, the mean time until each task executes again is $\mu/b = 1$, the mean comparison time of processing results of a job is $v/b = 1$, the mean time until a job executes again is $w/b = 1$, the probability that the comparison of processing results of a job agrees is $p = 0.8 \sim 1.0$.

Table 8.1 gives the optimal number N^* which minimizes $\ell_{0S}(N)$. For example, when $a/b = 200$ and $(1/\lambda)/b = 10800$, the optimal division number is $N^* = 3$. This indicates that N^* decreases with $(1/\lambda)/b$, however, increases with a/b, i.e., as the size of a job becomes large, N^* increases.

Table 8.2 gives the mean time $\ell_{0S}(N^*)$ when a job is divided into N^* tasks and $\ell_{0S}(1)$ when it is not divided. This indicates that the mean time $\ell_{0S}(N^*)$ decreases with $(1/\lambda)/b$ and p. From the comparison with mean times $\ell_{0S}(N^*)$ and $\ell_{0S}(1)$, it can be seen that the processing time becomes shorter about 15 percents by the division with signatures. In particular, the division is much effective in shortening the processing time when the size of a job is large.

8.5 Conclusions

From the viewpoint of accuracy and speeding-up of a job processing, we have investigated the reliability properties of a μP system where some errors are detected by signatures. We have derived the mean time and the total processing number of tasks until a job completes successfully. Further, we have discussed an optimal policy which minimizes the mean time.

From the numerical examples, we have shown the tendencies of the optimal division number which minimizes $\ell_{0S}(N)$ for various parameters, and that the division with signatures is effective when the size of a job is large.

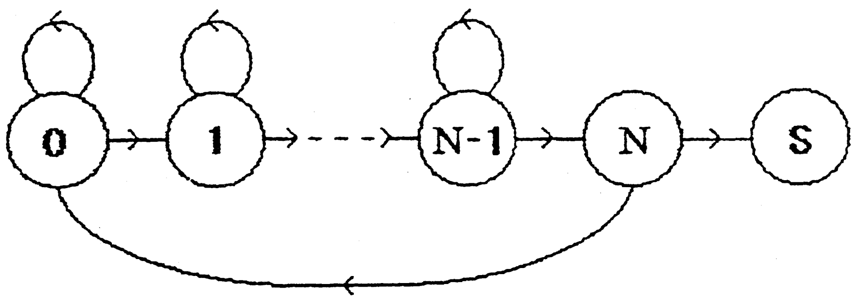


Figure 8.1: Transition diagram between system states.

Table 8.1: Optimal number N^* to minimize $\ell_{0S}(N)$.

a / b	(1/ λ) / b				
	3600	7200	10800	14400	18000
100	2	2	1	1	1
200	5	3	3	2	2
300	7	5	4	4	3
400	9	7	5	5	4

Table 8.2: Mean times $\ell_{0S}(N^*)$ and $\ell_{0S}(1)$.

a / b	p	(1/λ) / b				
		3600	7200	10800	14400	18000
100	0.8	135	131	130	130	129
		133	131	130	130	129
	0.9	120	117	116	115	115
		118	116	116	115	115
	1.0	108	105	104	103	103
		106	104	104	103	103
200	0.8	281	267	262	260	258
		264	260	258	258	257
	0.9	249	237	233	231	230
		234	231	230	229	228
	1.0	224	213	209	208	206
		211	208	207	206	205
300	0.8	440	409	399	393	390
		395	389	387	386	384
	0.9	392	364	354	350	347
		351	346	344	342	342
	1.0	352	327	319	315	312
		315	311	309	308	307
400	0.8	614	559	540	531	525
		526	518	515	513	512
	0.9	546	496	480	472	467
		467	461	458	456	455
	1.0	491	447	432	424	420
		420	415	412	411	410

Chapter 9

Conclusions

This thesis has studied the stochastic models of a microprocessor (μP) system. Using the theory of Markov renewal processes, we have obtained the reliability measures such as the mean times to system failure and to completion of the process. Moreover, we have derived expected costs and have analytically discussed optimal policies which minimize them. Finally, to understand the results easily, we have given numerical examples of each model and have evaluated them for various standard parameters. If some parameters are estimated from actual data, we could select the best policy.

In Chapter 2, we have considered a μP system with a watchdog timer (WDT) which is preventively maintained at time T and at reset number N. The availability of the system has been obtained, and an optimal inspection time and reset number which maximize it have been discussed. It has been shown from the numerical examples that the coverage of a WDT plays an important role for providing the system with high reliability.

In Chapter 3, we have treated a system where a main processor (MPu) has N watchdog processors (WDPs) with self-checking. To show the number of WDPs for prevention that the MPu becomes faulty, we have formulated the model where the system has N standby redundant WDPs. The reliability function and the expected cost until the MPu becomes faulty have been derived, and an optimal number of WDPs which minimizes the expected cost has been analytically discussed. It has been shown that it is effective to have at least one WDP when the system requires a high reliability.

In Chapter 4, we have studied a system with N μP units, where each μP unit consists of μP and WDP. Under the assumption that a μP is in faulty state if more than K resets have occurred at time T, we have derived the mean time until system failure. Introducing the cost of a μP, we have analytically discussed the problem to obtain how many number of μP units is optimal. It has been shown numerically that the system is enough to have only one unit when the reset number K takes ordinary values from 4 to 8.

From the viewpoint of real-time processing of the system, it would be necessary to have the function which completes one processing within a certain limit time. In Chapter 5, we have discussed the model of a system with N μP units. Under the assumption that a μP is in faulty state if it does not finish one processing until a limit time T, we have obtained the mean time and the mean processing number until system failure. Moreover, we have derived the cost effectiveness and have discussed an optimal number of μPs which minimizes it. An interesting consequence has been obtained numerically that when a limit processing time is small comparatively, as the μP unit becomes advanced, the expected cost per unit of processing decreases, and oppositely, the optimal number increases.

In Chapter 6, we have considered a system with N TMR (Triple Modular Redundancy) units in which each unit consists of μP and WDP. Introducing the concept of complexity, the mean time to system failure and the expected cost have been derived, and optimal numbers of TMR units which maximize or minimize them have been analytically discussed. It has been found that to develop the reliability of the system, we should more improve the coverage of a WDP.

In Chapter 7, we have dealt with the problem for improving the reliability of a μP system with network processing, and have derived the mean time and the expected reset number until a network processing is successful. Further, we have analytically discussed an optimal reset number which minimizes the expected cost. It has been shown that when errors of a μP do not occur frequently, the optimal reset number is almost determined by the cost rate of an interruption of processing.

The reliability problem of a μP system whose errors can be detected by using signatures has been proposed in Chapter 8. We have derived the mean time and the total processing number of tasks until a job completes successfully. Further, we have discussed an optimal division number of a job. It has been shown from the numerical examples that the division with signatures is effective when the size of a job is large.

As VLSI (Very Large Scale Integration) technology has rapidly developed, μPs have been used in many actual areas. It would be very important to evaluate and improve the reliability of systems with μPs. The results obtained in this thesis would be applied to practical fields by making some suitable modifications and extensions. As examples, Chapters 3, 4, 5 and 6 could be applied to not only the system of automobiles but also the systems of space rockets and deep sea explorations, which cannot undergo corrective maintenances by repairmen. Further, Chapter 2 would be applicable to the following policies: (i) The error detection policy of ROM (Read Only Memory) programming on a design and development stage, (ii) the preventive maintenance and replacement policies of a μP on an operational stage, and (iii) the policy to improve mission availability when an operational time is given.

Finally, we enumerate the following questions for future studies:

(1) Is it possible to estimate statistically various parameters in the formulated models?

(2) What types of distribution are fit for the observed data?

(3) What are appropriate measures which show the reliability of the system?

Various kinds of larger and more complicated systems will be grown up in future industries. We also would consider and formulate new stochastic models, and analyze their characteristics and evaluate their performances, using the techniques and the results of this thesis. Further studies for such subjects would be greatly expected.

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Chapter 4

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